



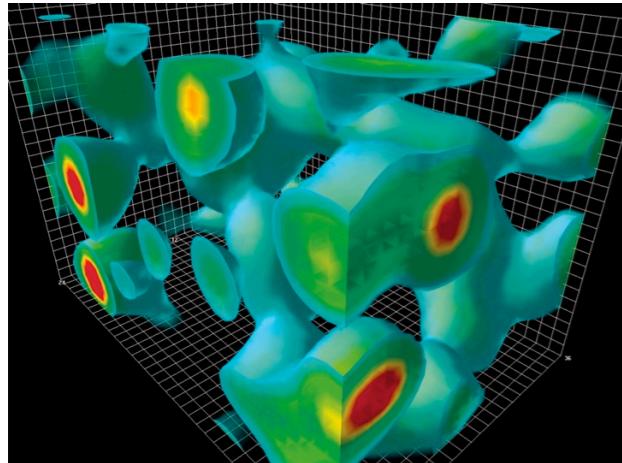
# Nuclear Physics from Lattice QCD

The Anticipated Impact of Exa-scale Computing

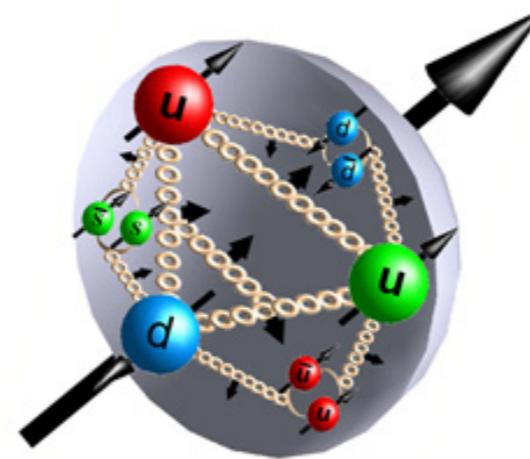
Martin J. Savage  
University of Washington  
December 2010, LBNL

# The Scope and Impact of Nuclear Physics

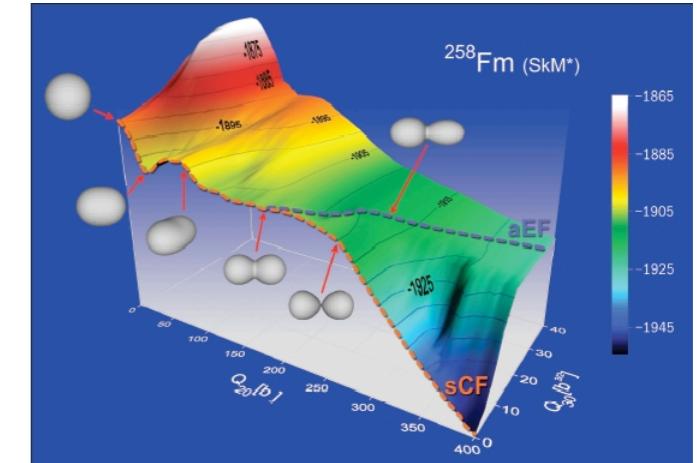
QCD



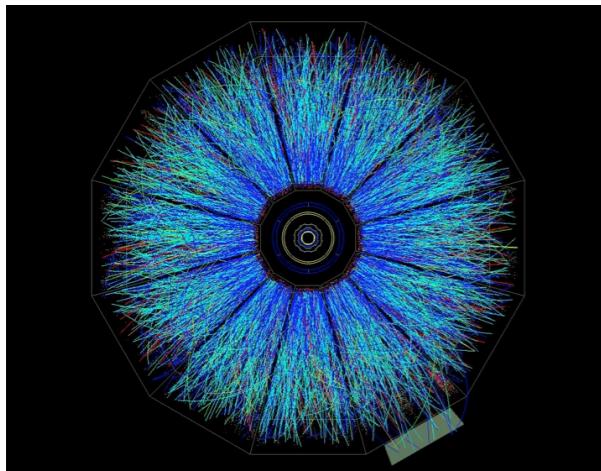
Structure of Nucleons



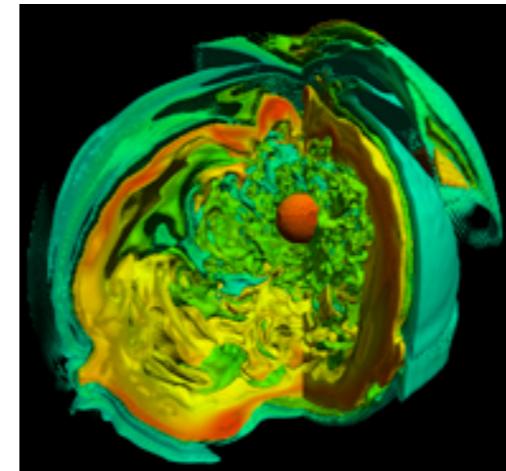
Nuclei



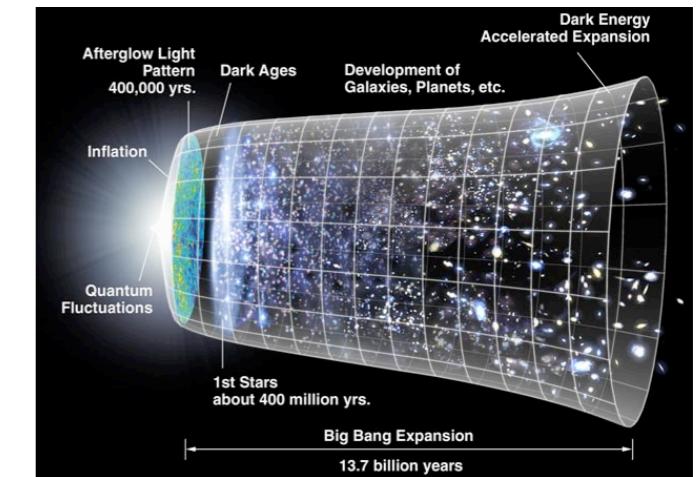
Extreme Conditions



Elements



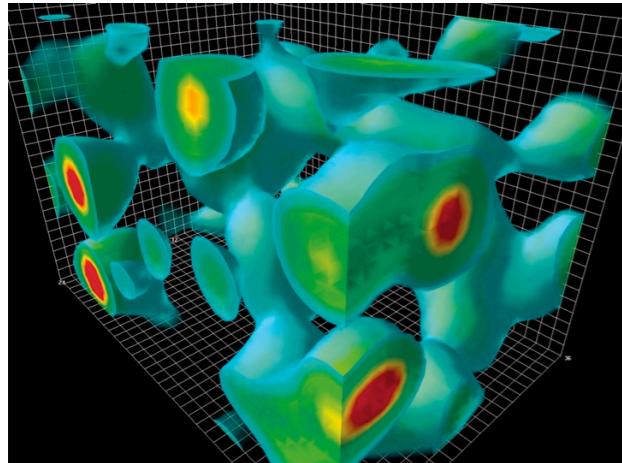
Central Role in Cosmology



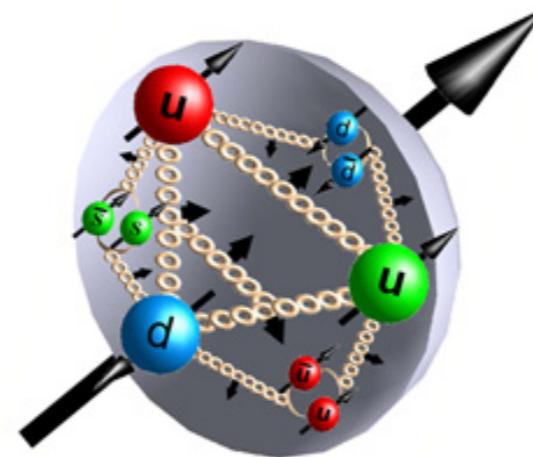
- Enormous range of length-scales
  - Exciting, difficult
  - Requires diverse analytic and numerical methods

# The Scope and Impact of Nuclear Physics

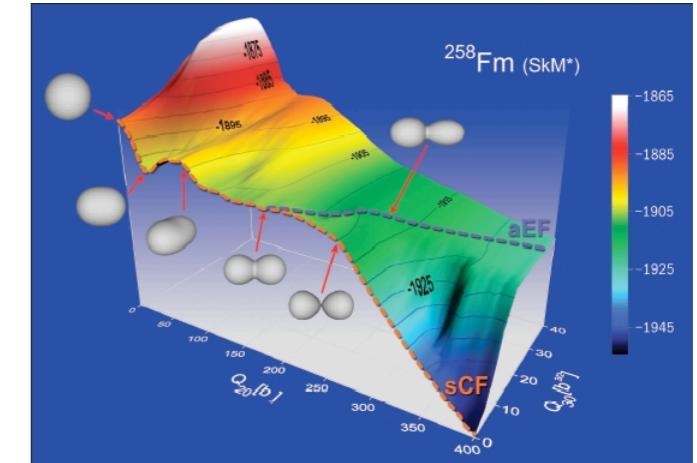
QCD



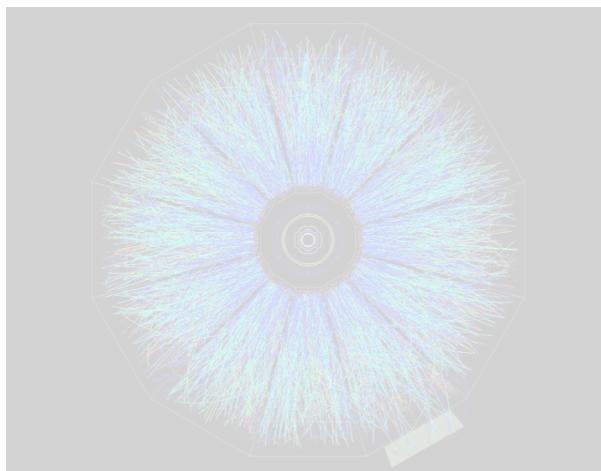
Structure of Nucleons



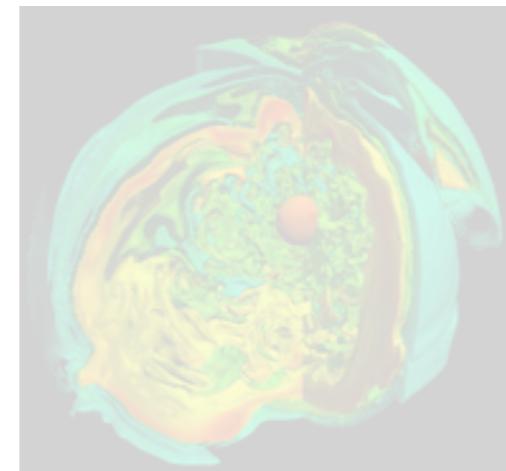
Nuclei



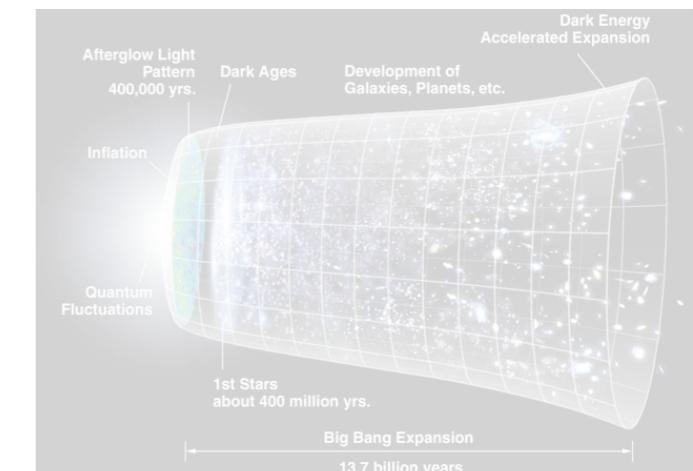
Extreme Conditions



Elements



Central Role in Cosmology

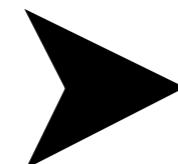
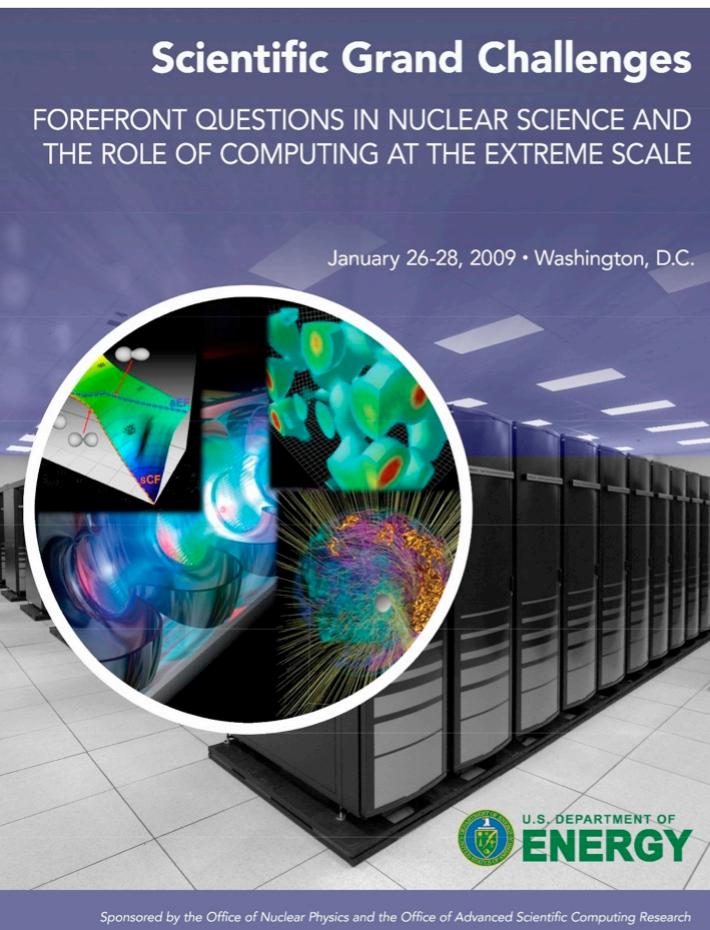


- Enormous range of length-scales
  - Exciting, difficult
  - Requires diverse analytic and numerical methods

# Exa-Scale Computing Workshops

## 2009-2010

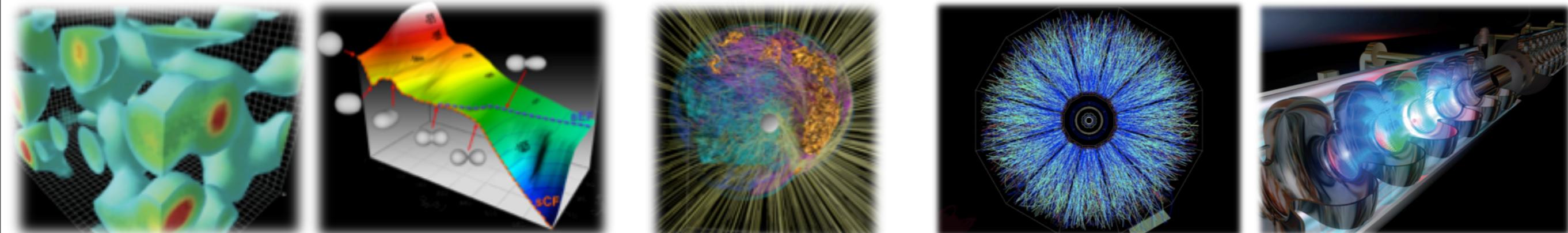
$$1 \text{ Exaflop} = 10^3 \text{Petaflops} = 10^6 \text{Teraflops} = 10^9 \text{GigaFlops}$$



Architectures and Technology  
Biology  
Basic Energy Sciences  
Climate Science  
Cross-Cutting Workshop  
Fusion Energy  
High Energy Physics  
National Security  
Nuclear Energy  
Nuclear Physics

( Trivelpiece committee )

# Exa-Scale Computational Resources



Cold QCD and  
Nuclear Forces

Nuclear Structure  
and Reactions

Nuclear  
Astrophysics

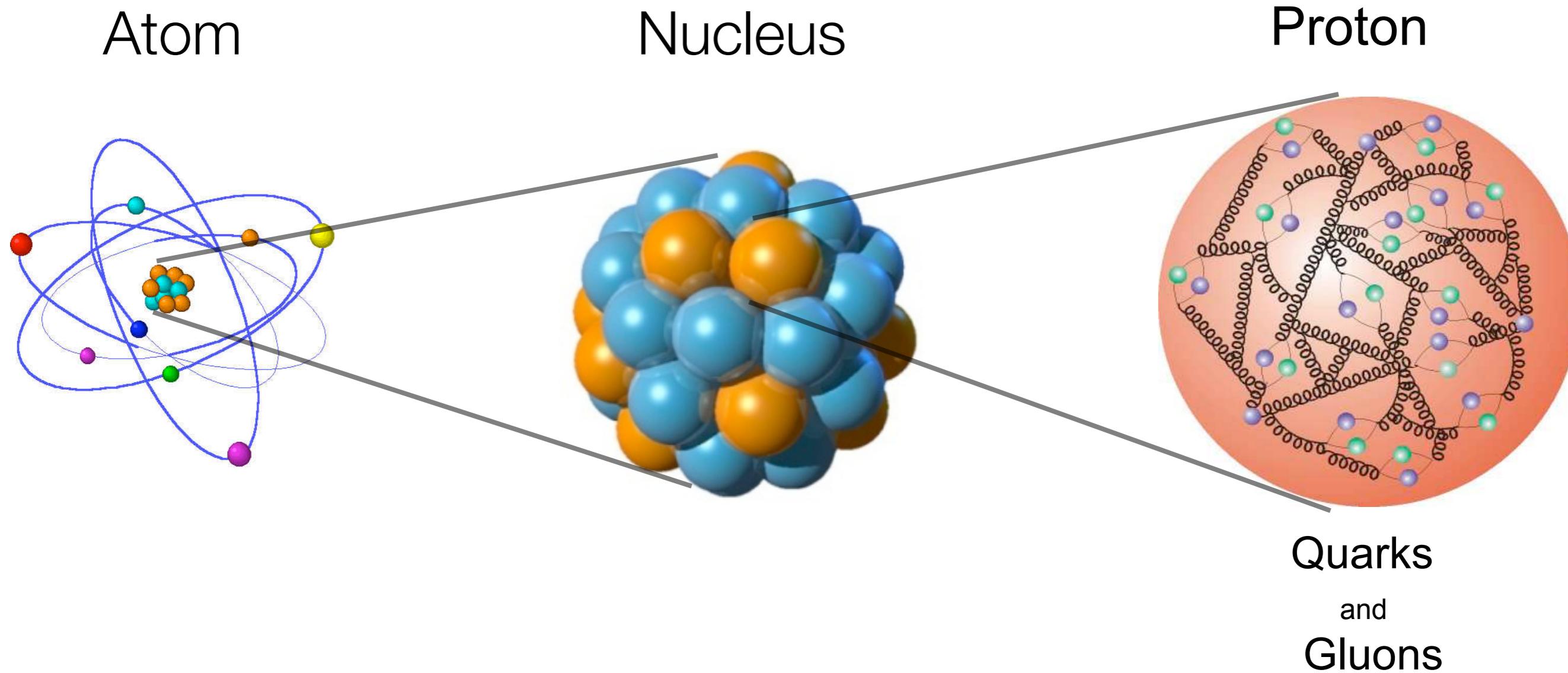
Hot and Dense  
QCD

Accelerator  
Physics

- **Predictive Capability for Nuclear Physics:** Calculations with **quantifiable uncertainties** of processes occurring in nuclei, and in dense and hot matter where experiments are not possible.

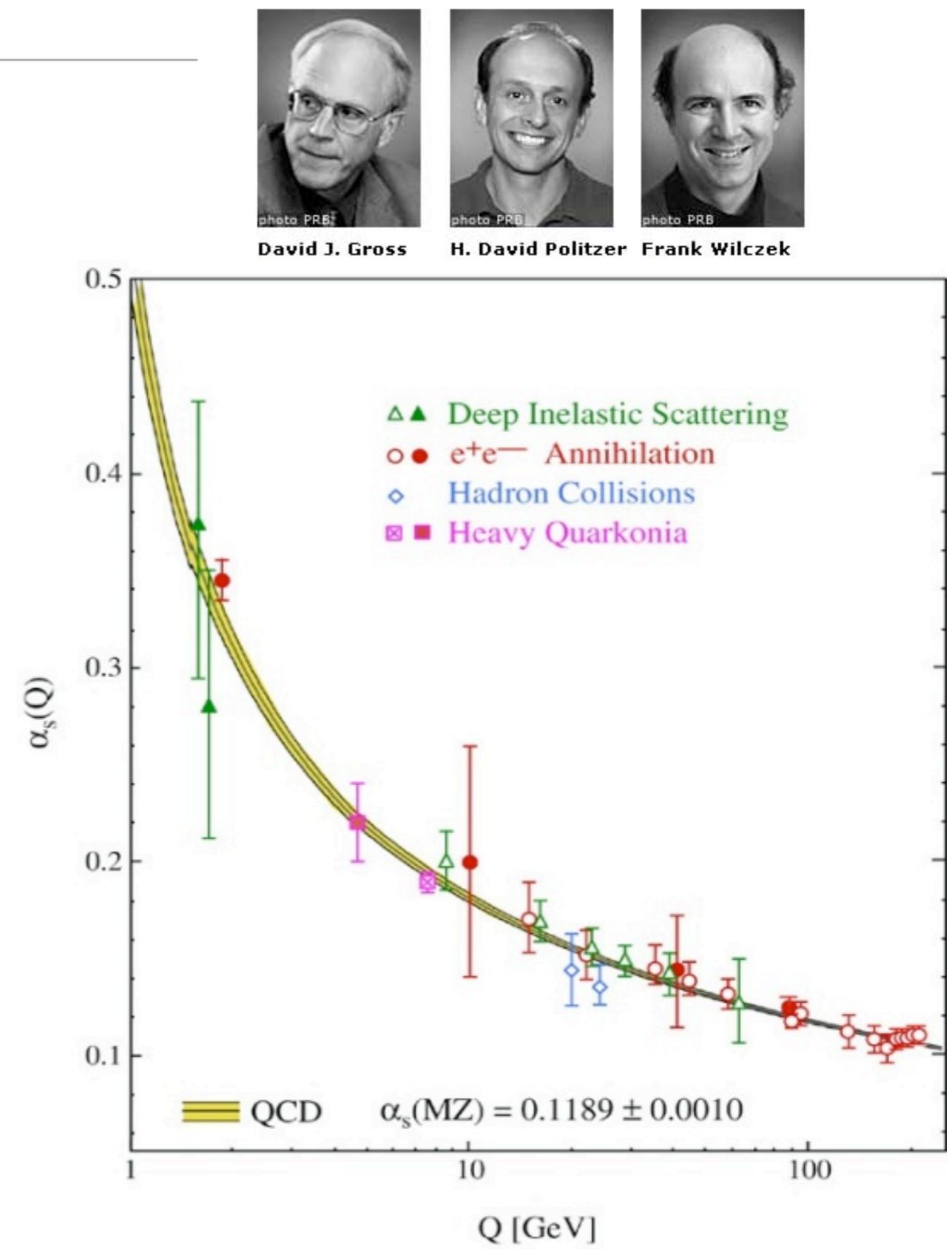
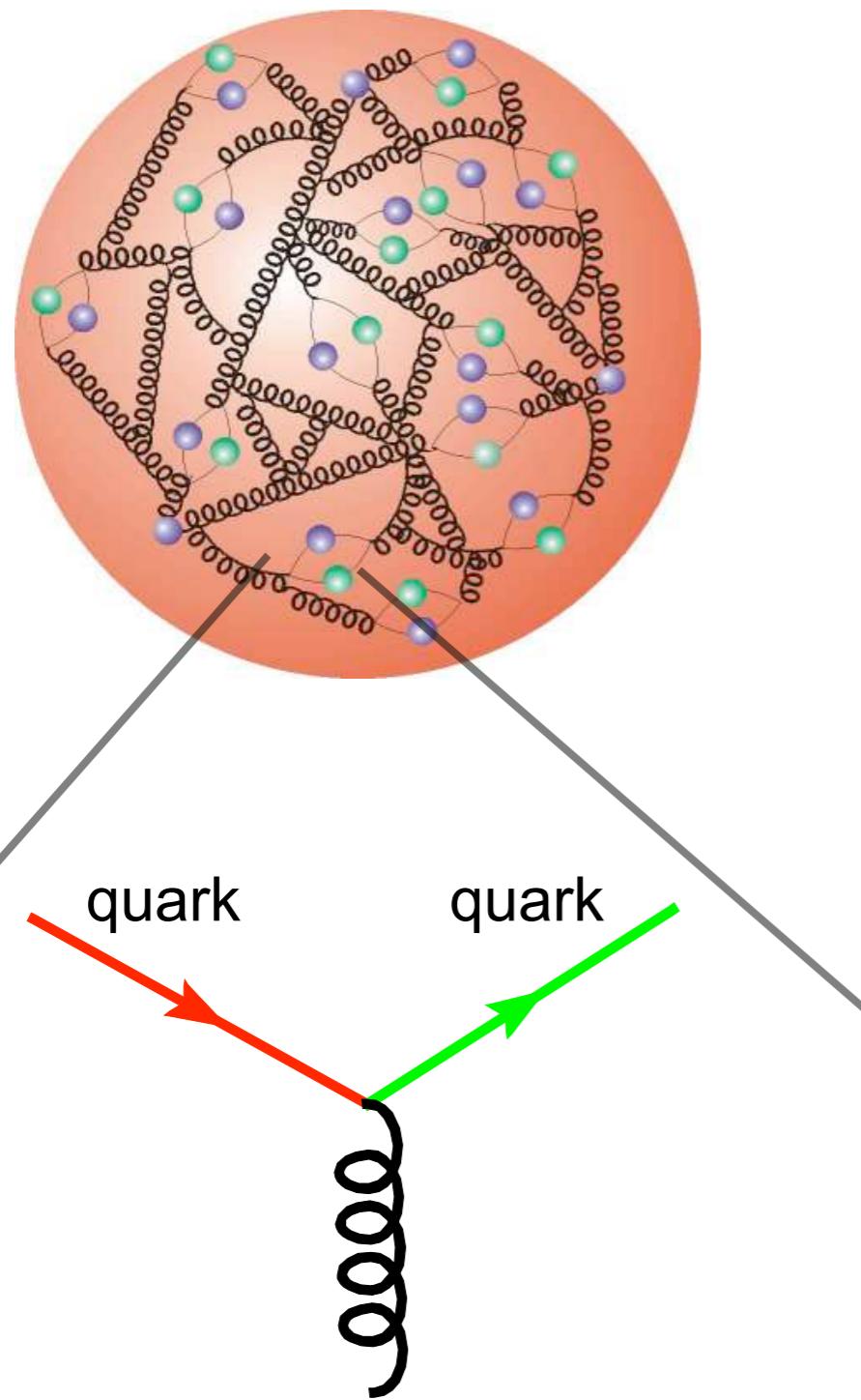
Exa-scale Computing is required to accomplish the objectives of the Nuclear Physics Research Program

# The Structure of Matter



Direct Connection to Fundamental Physics

# Quantum Fluctuations make the Strong Interaction Strong



David J. Gross



H. David Politzer



Frank Wilczek

The QCD Vacuum is Complicated  $\hbar \sim 10^{-34}$  Js

---

Action Density

Quantum Fluctuations of  
the Gluon Fields



$2.5 \times 10^{-15}$  m

$\Delta t \sim 10^{-23}$  s

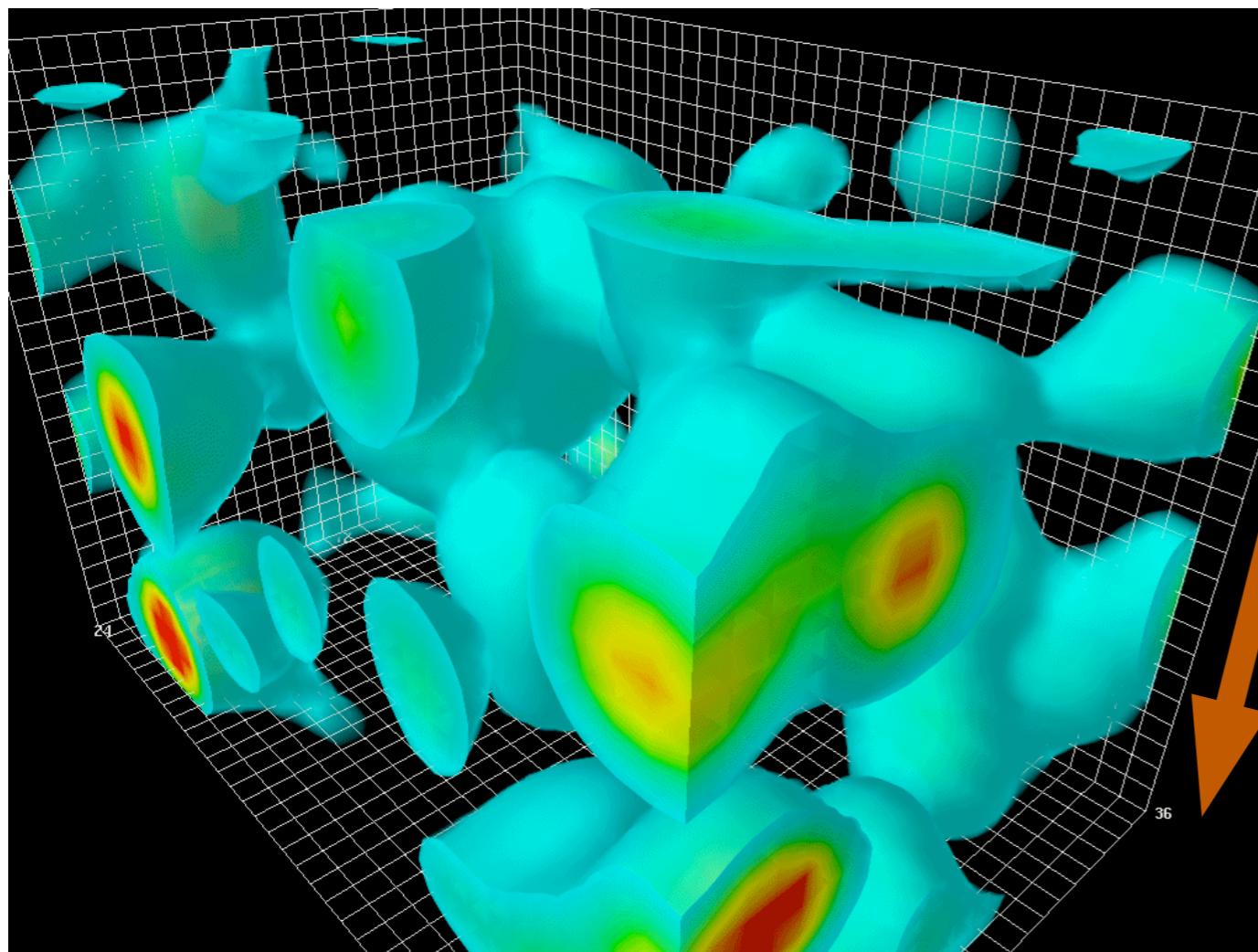
(Derek Leinweber, U. of Adelaide)

Responsible for most of the known mass

# The QCD Vacuum is Complicated $\hbar \sim 10^{-34}$ Js

---

Action Density



(Derek Leinweber, U. of Adelaide)

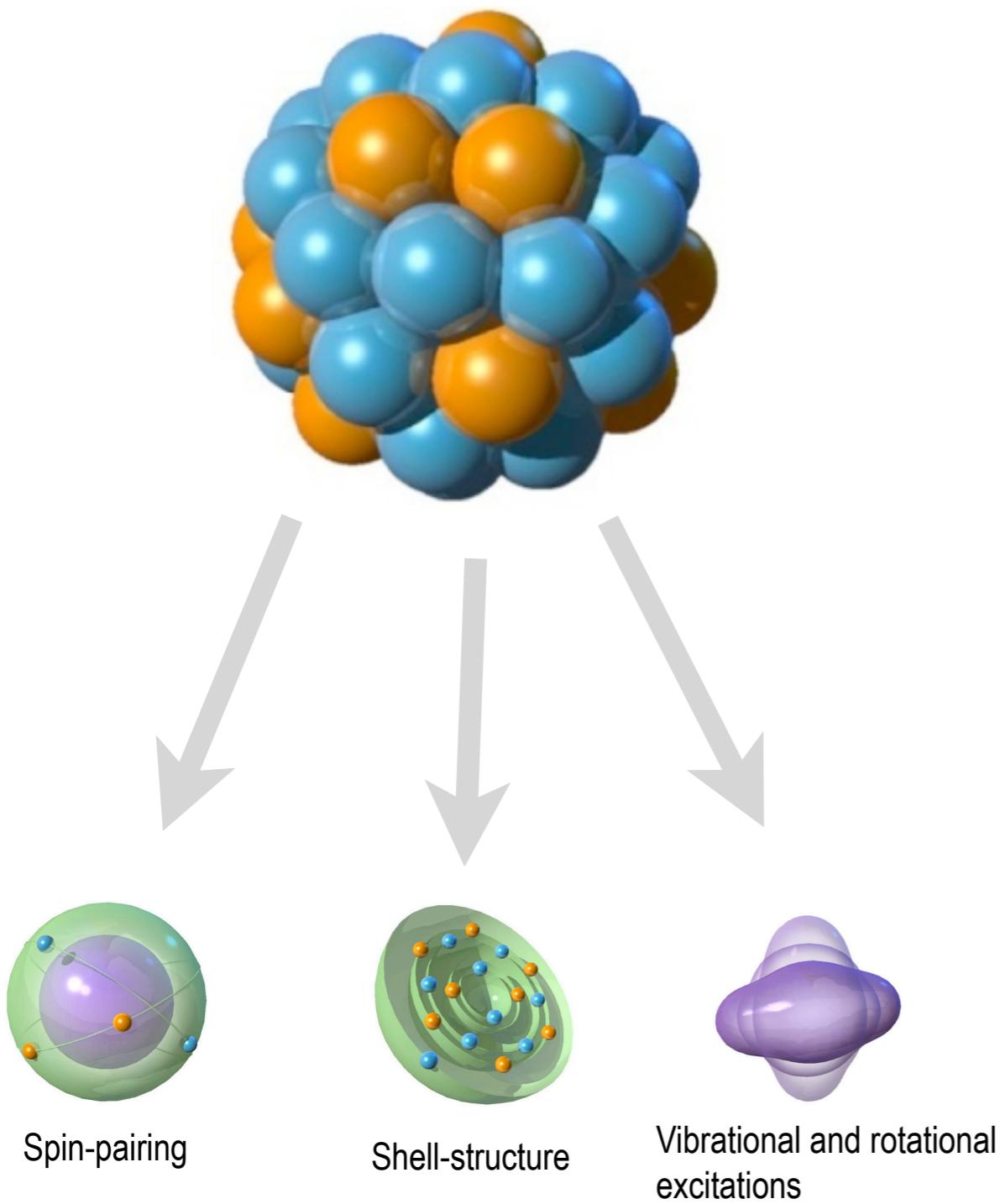
Quantum Fluctuations of  
the Gluon Fields

$$2.5 \times 10^{-15} \text{ m}$$

$$\Delta t \sim 10^{-23} \text{ s}$$

Responsible for most of the known mass

# Nuclei and QCD



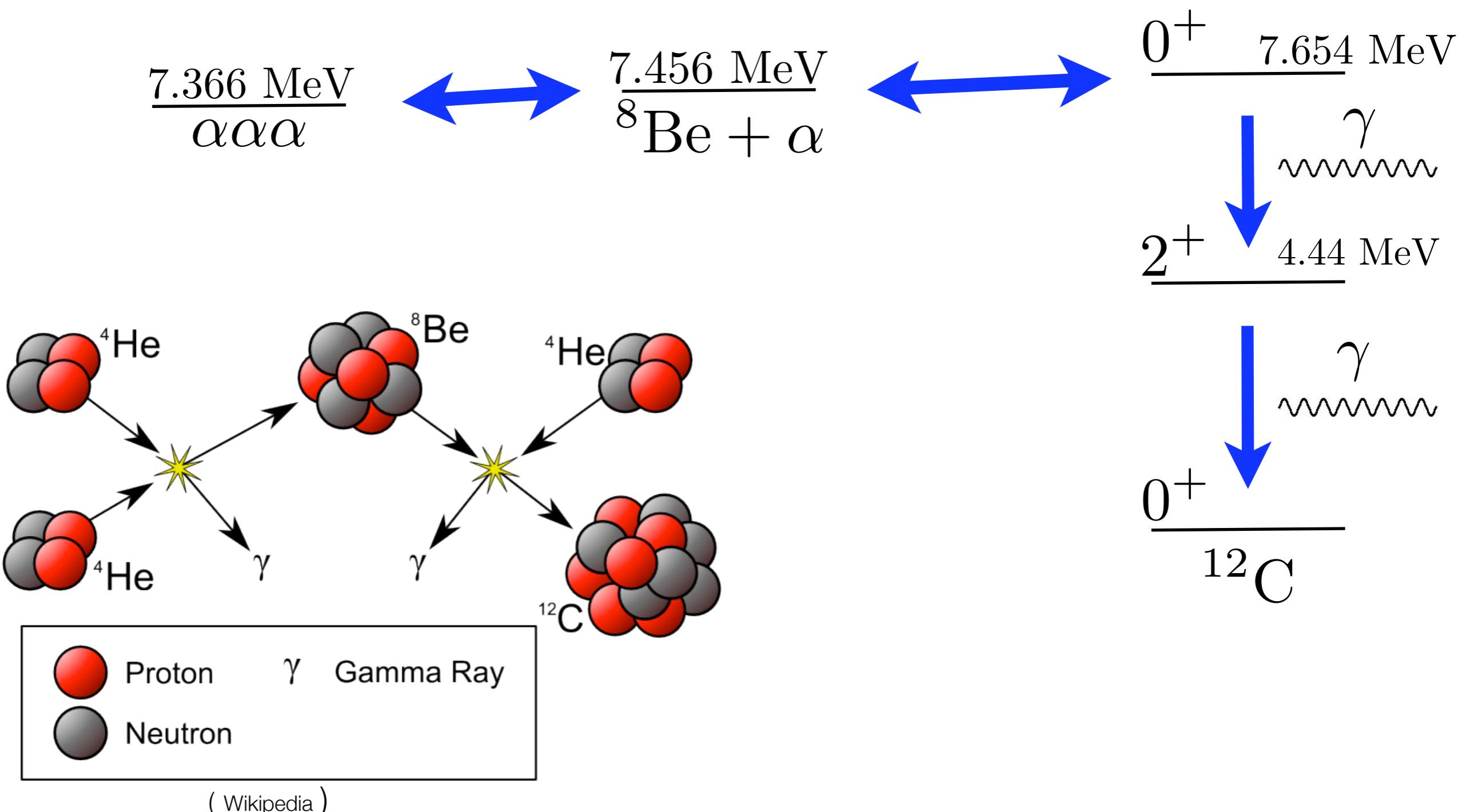
$\Lambda_{\text{QCD}}$

$$\frac{m_u}{\Lambda_{\text{QCD}}} \quad \frac{m_d}{\Lambda_{\text{QCD}}} \quad \frac{m_s}{\Lambda_{\text{QCD}}}$$

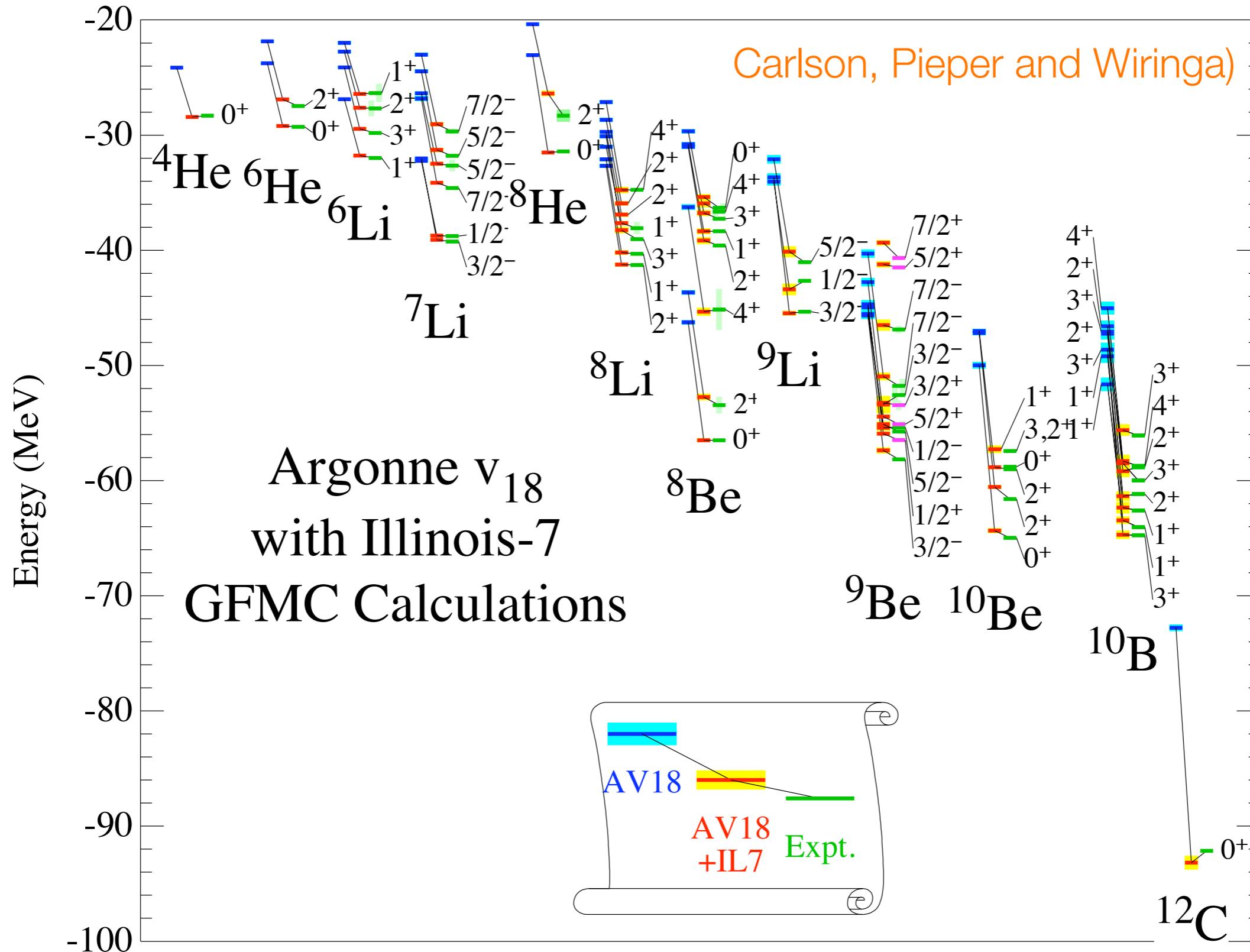
$\alpha_e$

# Fine Tunings in Nuclear Physics

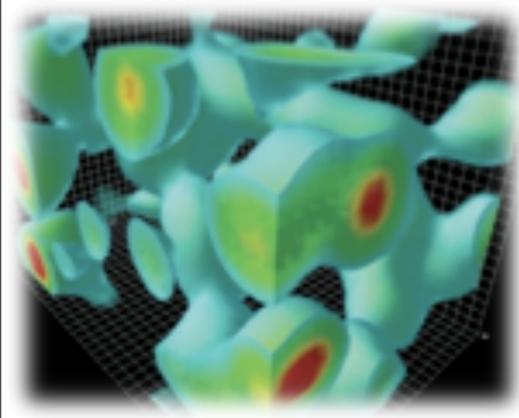
## - Carbon Production in Stars



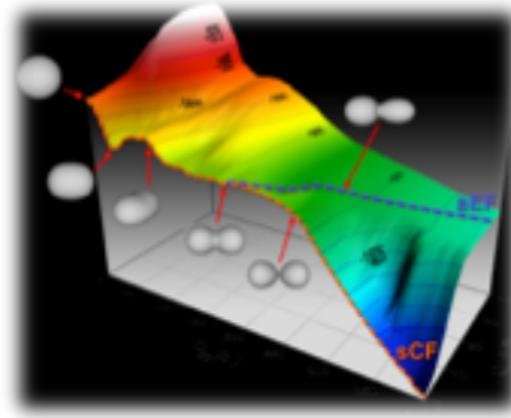
# Nuclei and Nuclear Interactions



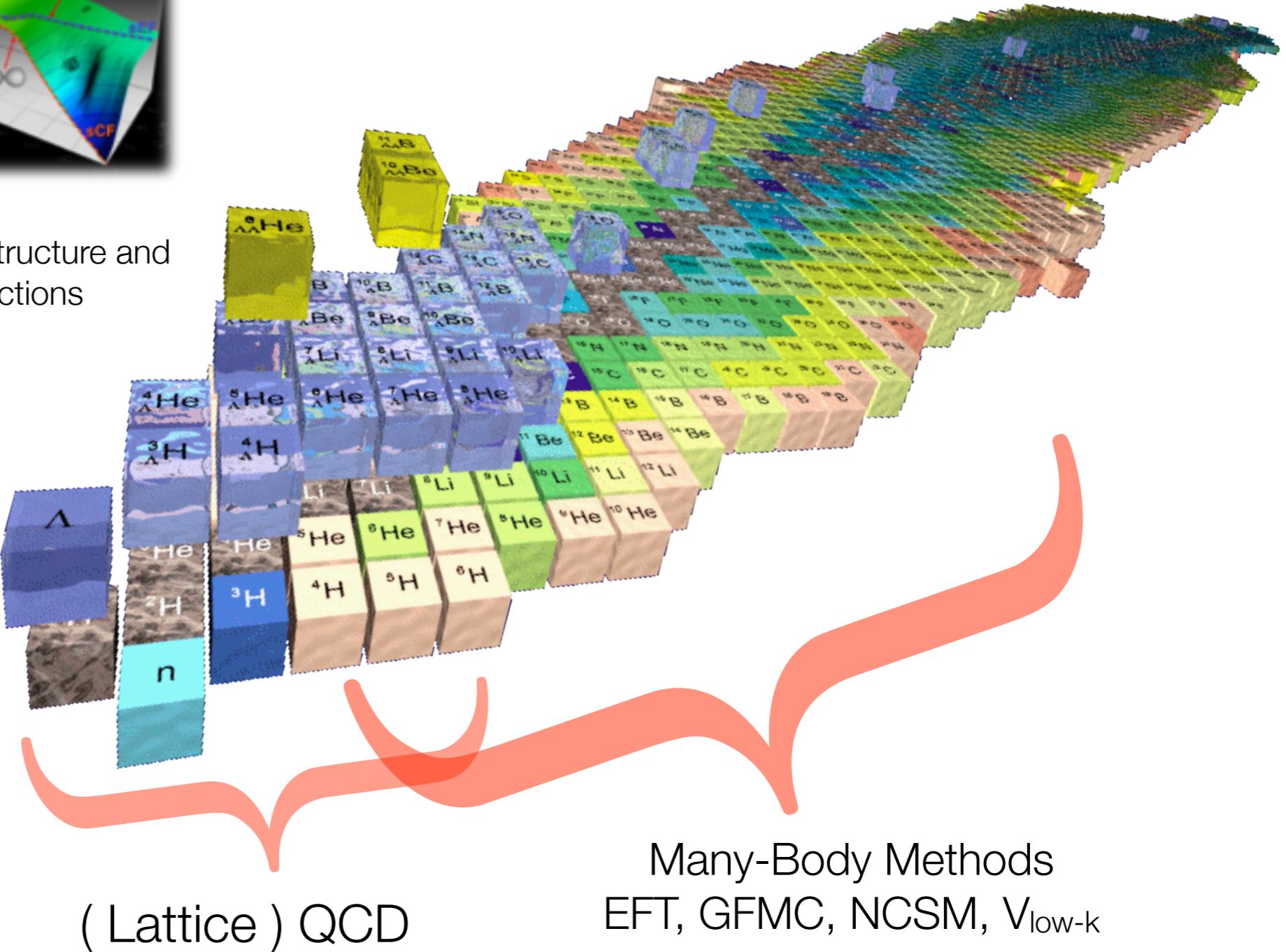
# ( Partial ) Unification of Nuclear Physics

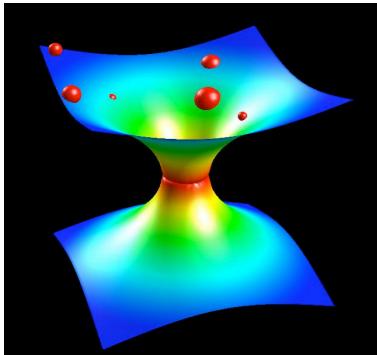


Cold QCD and  
Nuclear Forces



Nuclear Structure and  
Reactions



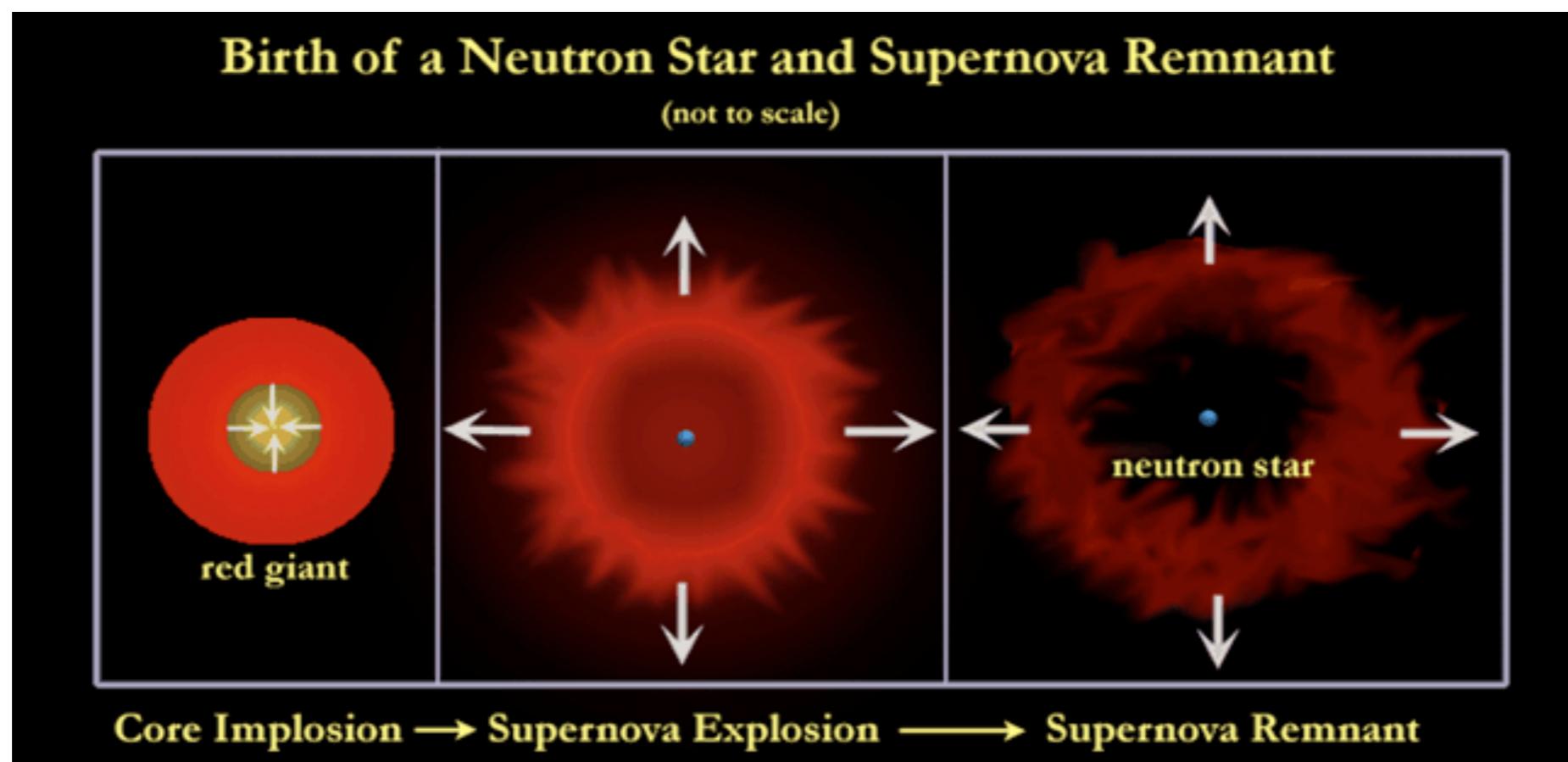


# Dense Matter and Astrophysics

Equation of State of Nuclear Material at High Densities ??

- Kaon (strange meson) Condensate ?
- Sigma (strange) Baryons ?

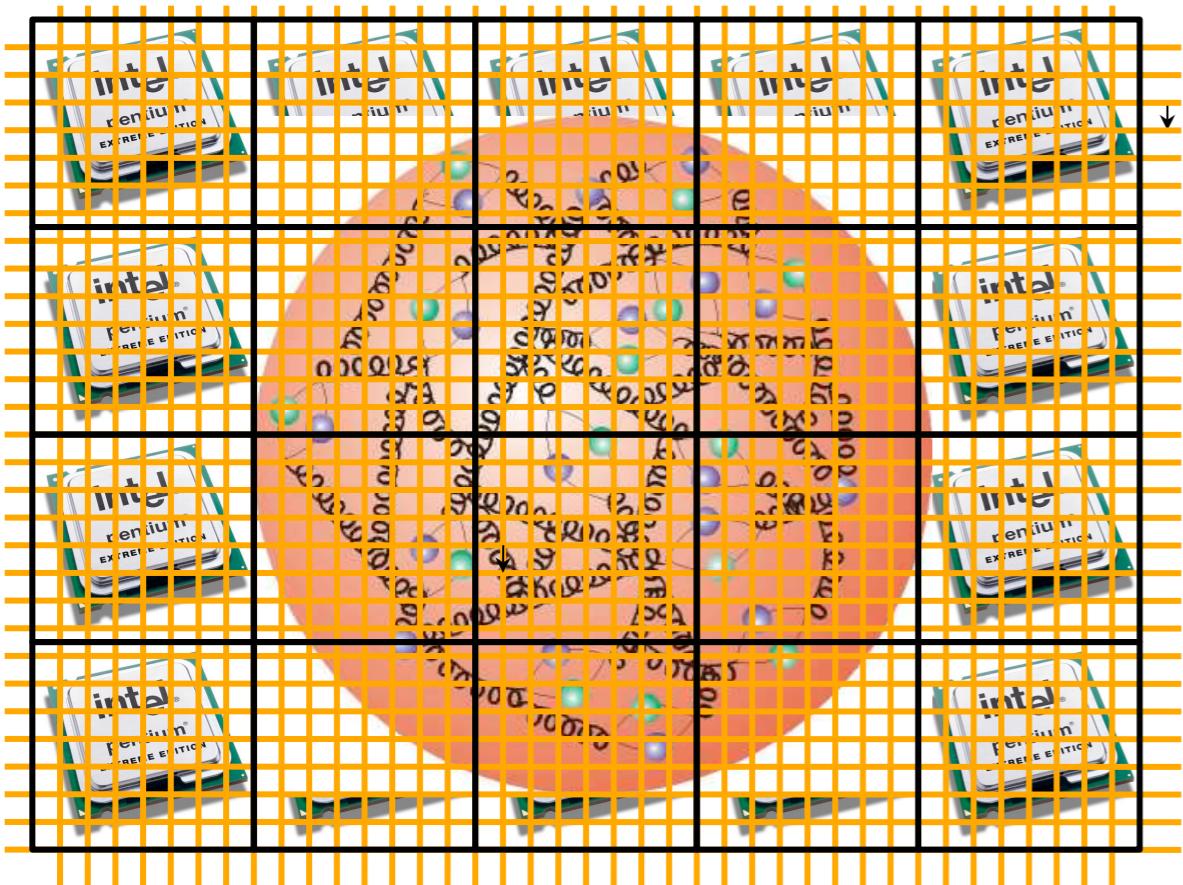
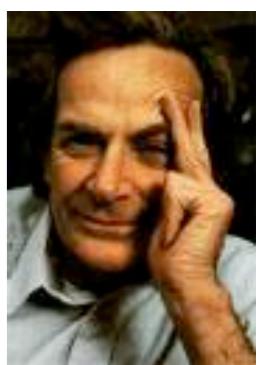
Is SN1987A a Black Hole or a Neutron Star ?





# Lattice QCD :

## Numerical Evaluation of QCD Path Integral



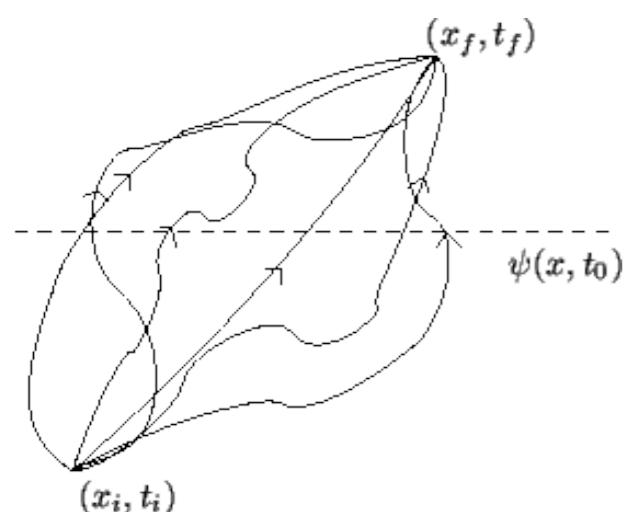
Lattice Spacing :  
 $a \ll 1/\Lambda\chi$

Lattice Volume :  
 $m_\pi L \gg 2\pi$

Effective Field Theory gives form of  
extrapolation  $a = 0$  and  $L = \infty$

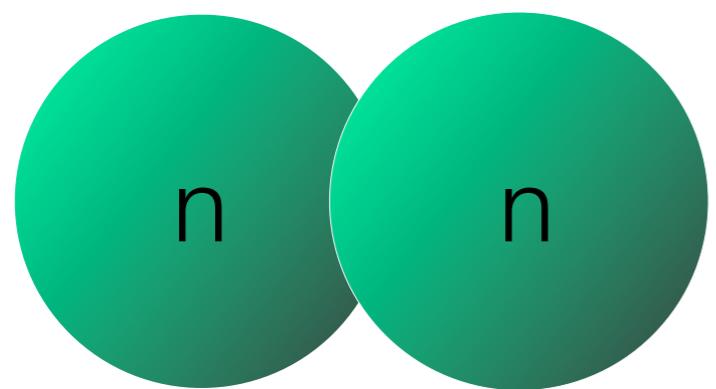
e.g. triton correlation function

$$\langle \hat{\theta} \rangle \sim \int \mathcal{D}\mathcal{U}_\mu \hat{\theta}[\mathcal{U}_\mu] \det[\kappa[\mathcal{U}_\mu]] e^{-S_{YM}}$$
$$\rightarrow \frac{1}{N} \sum_{\text{gluon cfgs}}^N \hat{\theta}[\mathcal{U}_\mu]$$

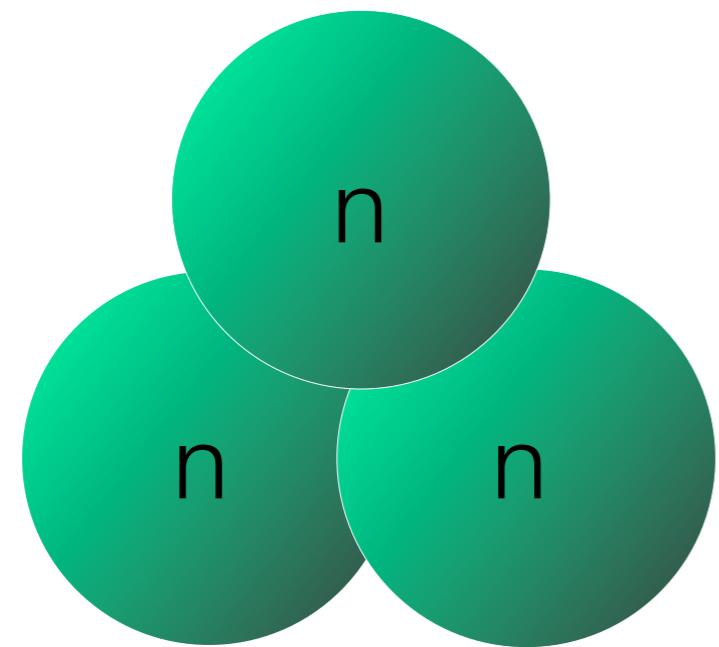


# Nuclear Interactions

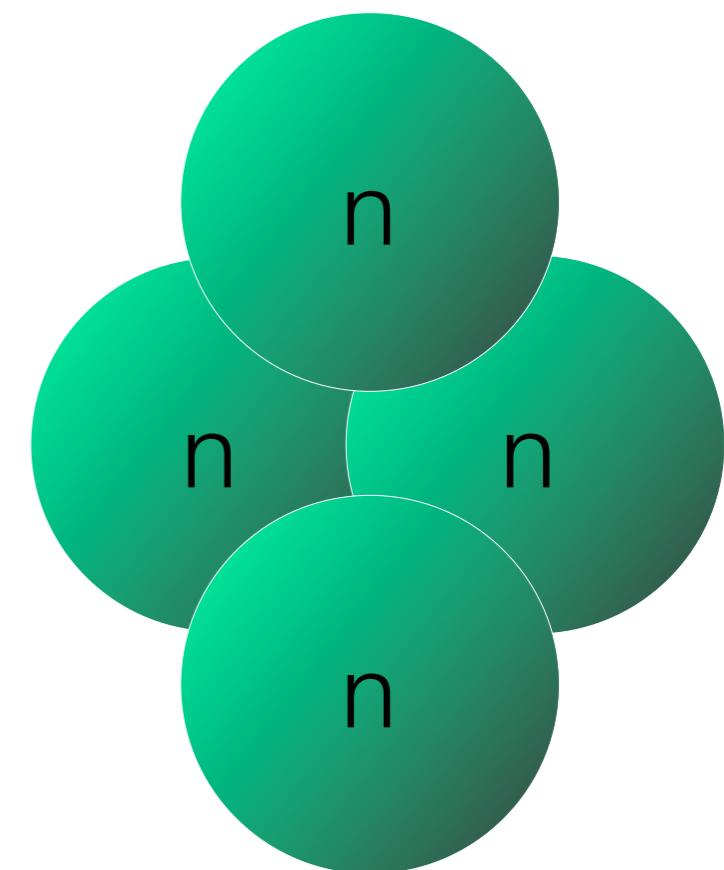
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NN-interaction -- verification



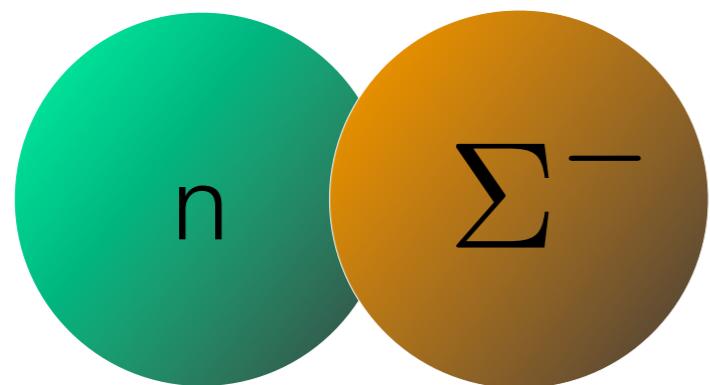
NNN



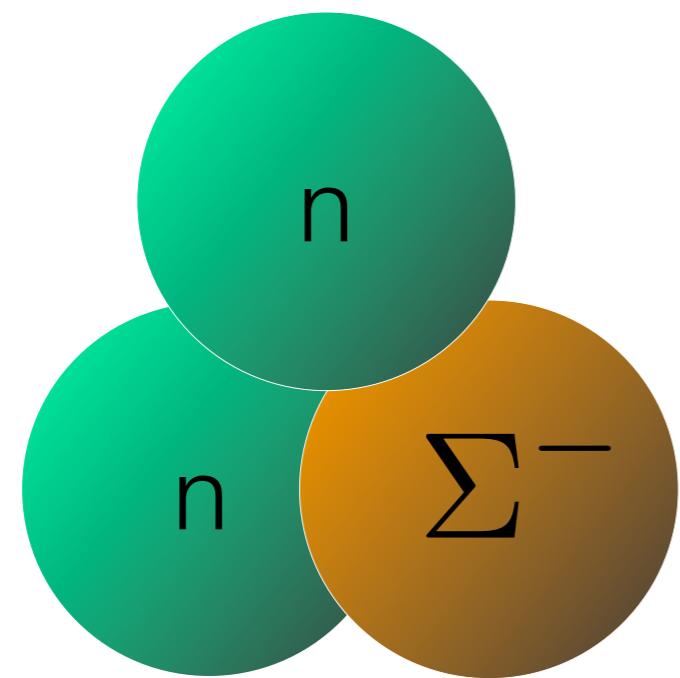
NNNN

# Hyperon-Nuclear Interactions

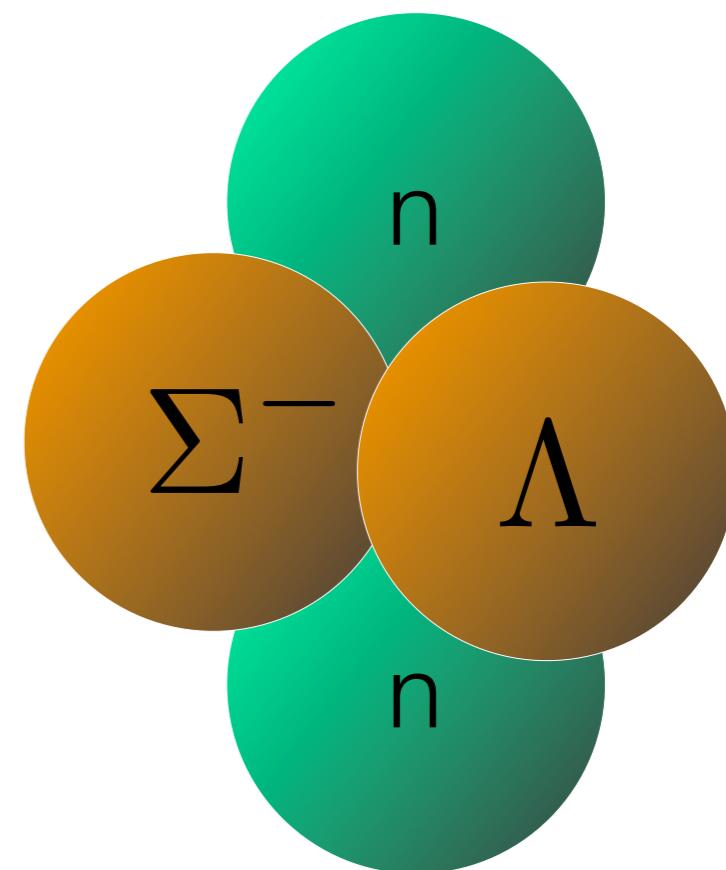
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YN-interaction

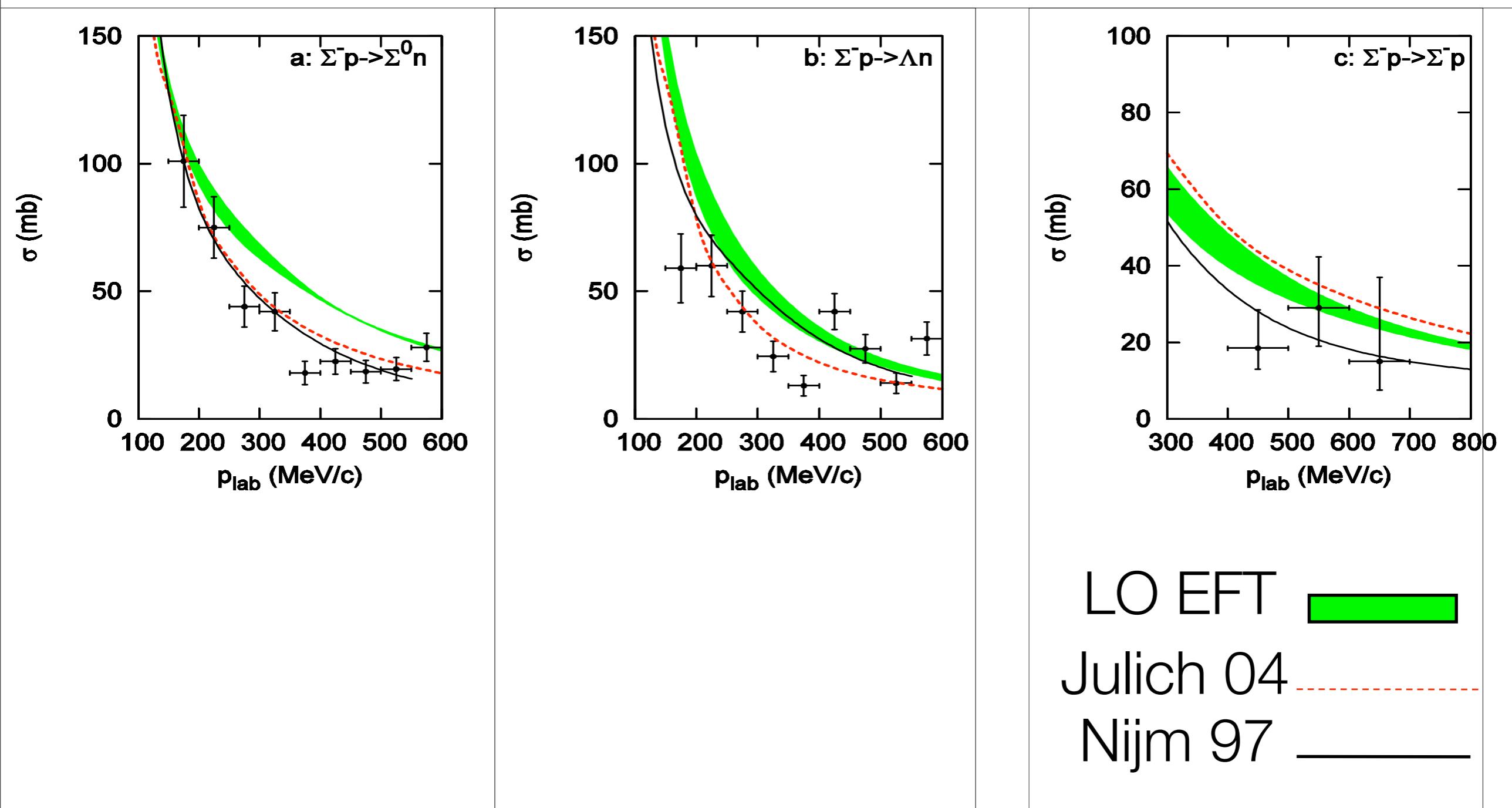


YNN



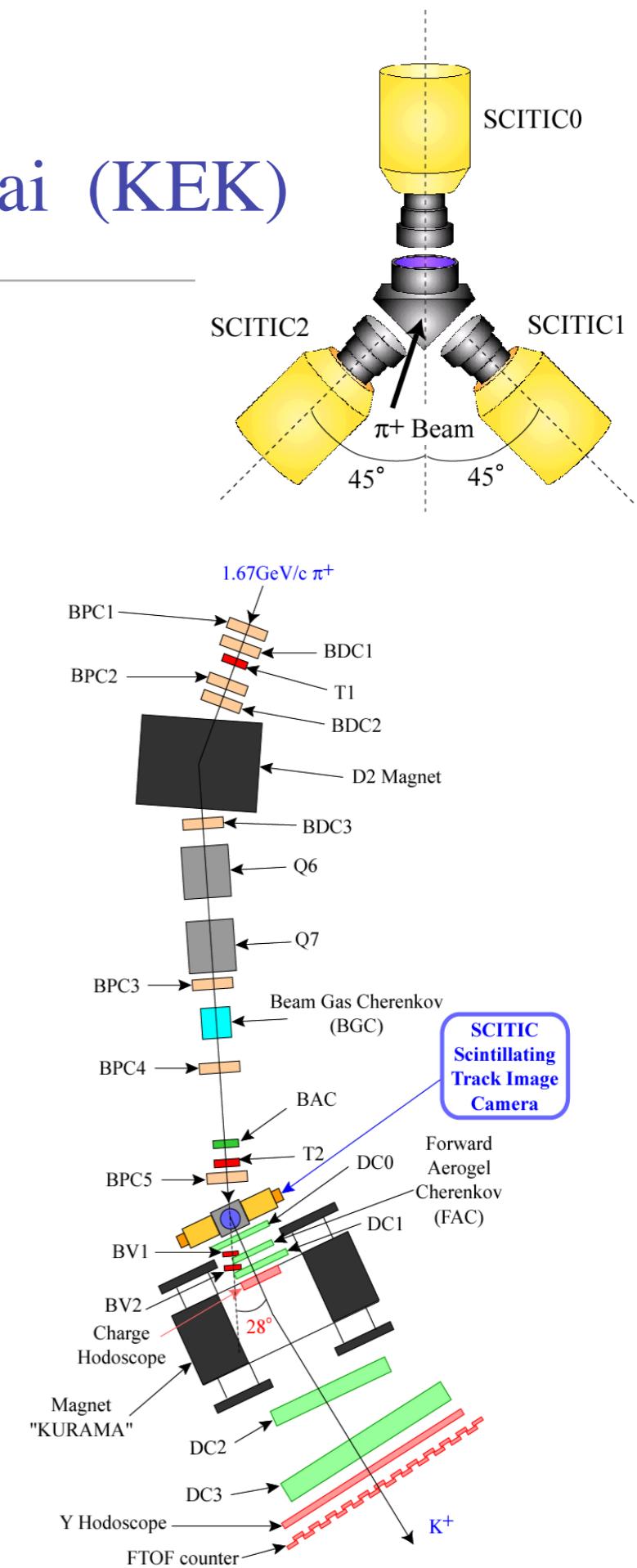
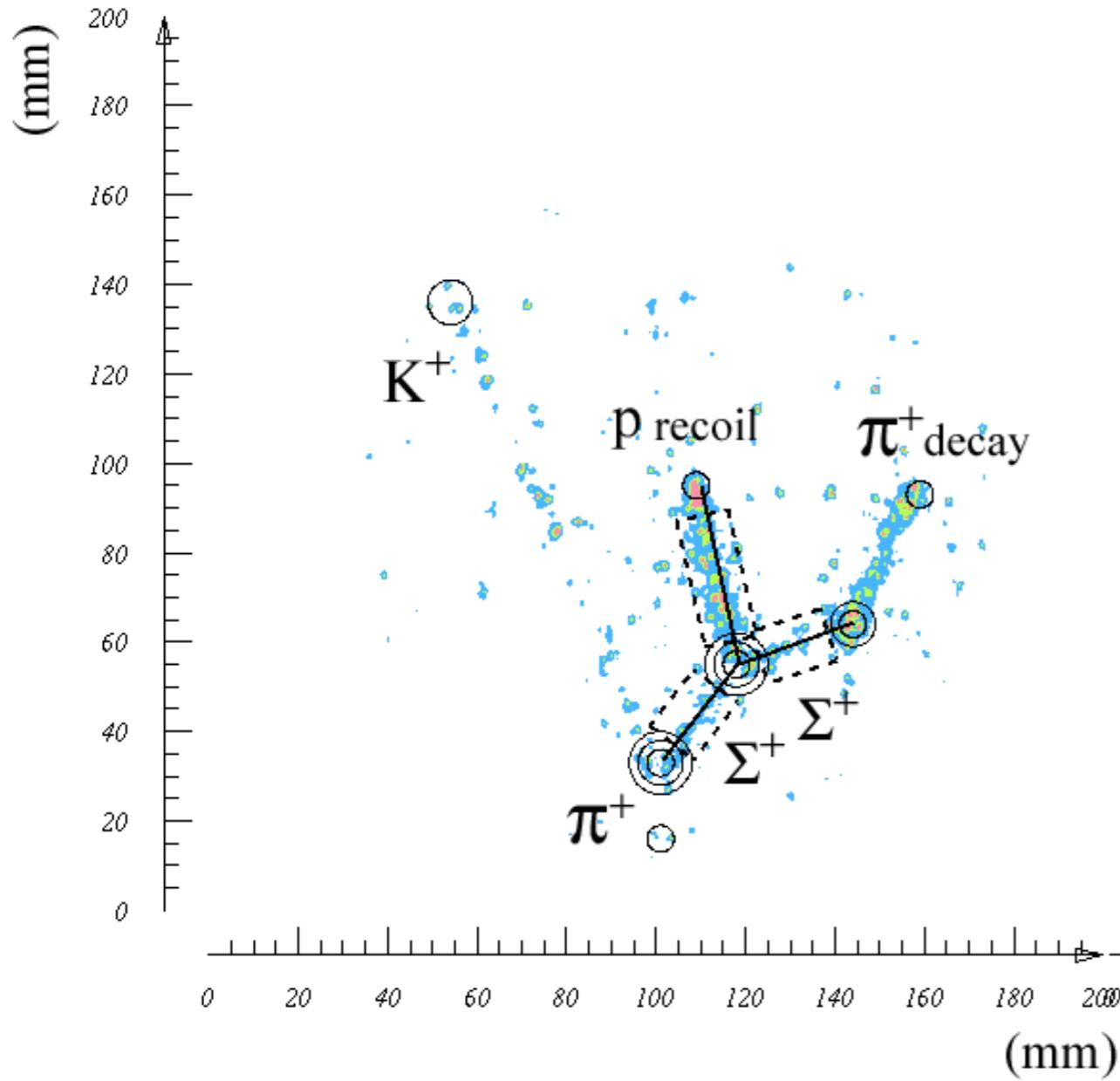
YYNN

# Some of the Experimental Hyperon-Nucleon Data

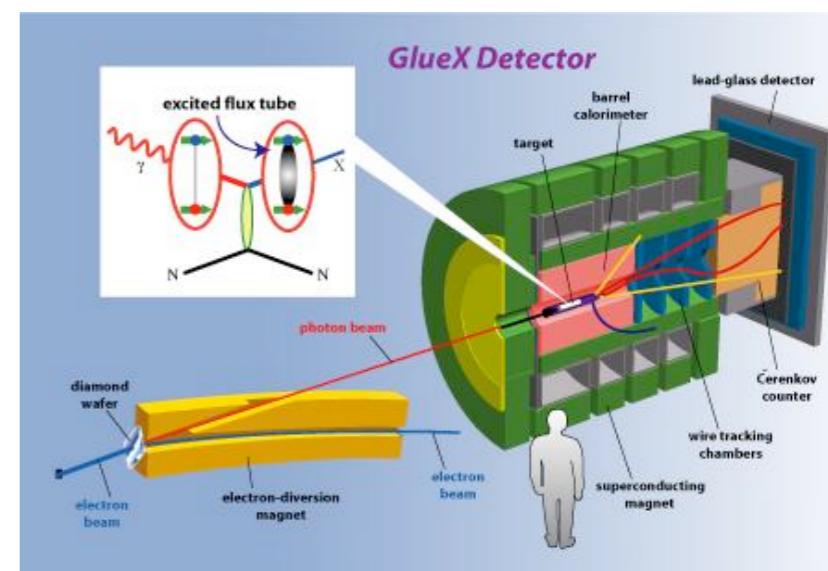
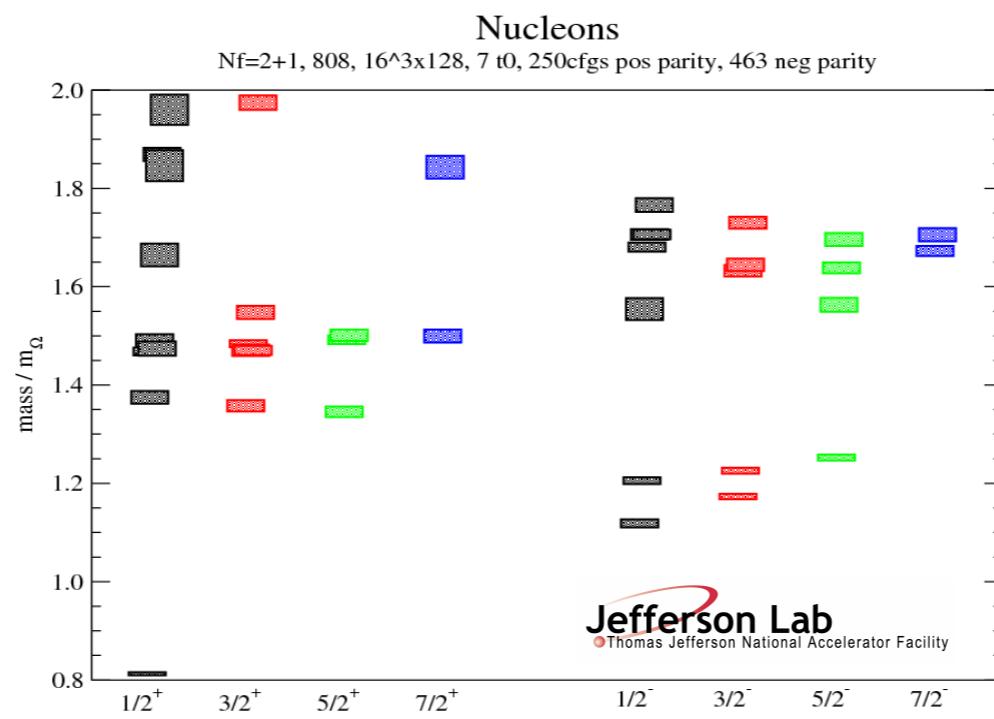
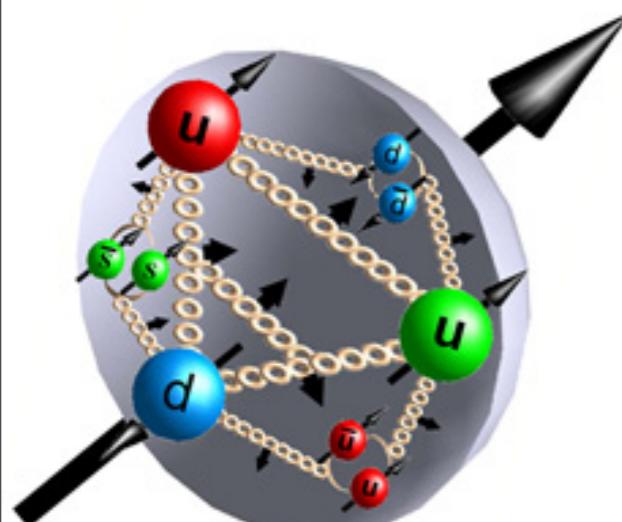
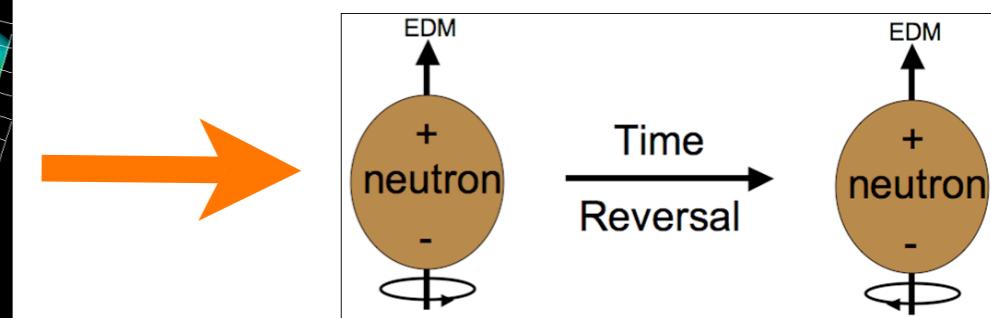
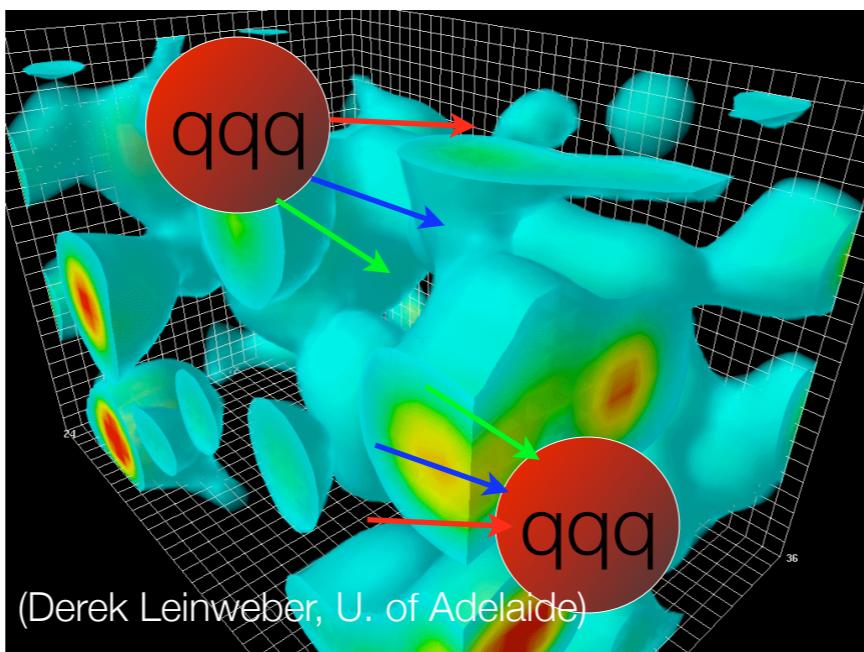


# YN : Experimental Situation

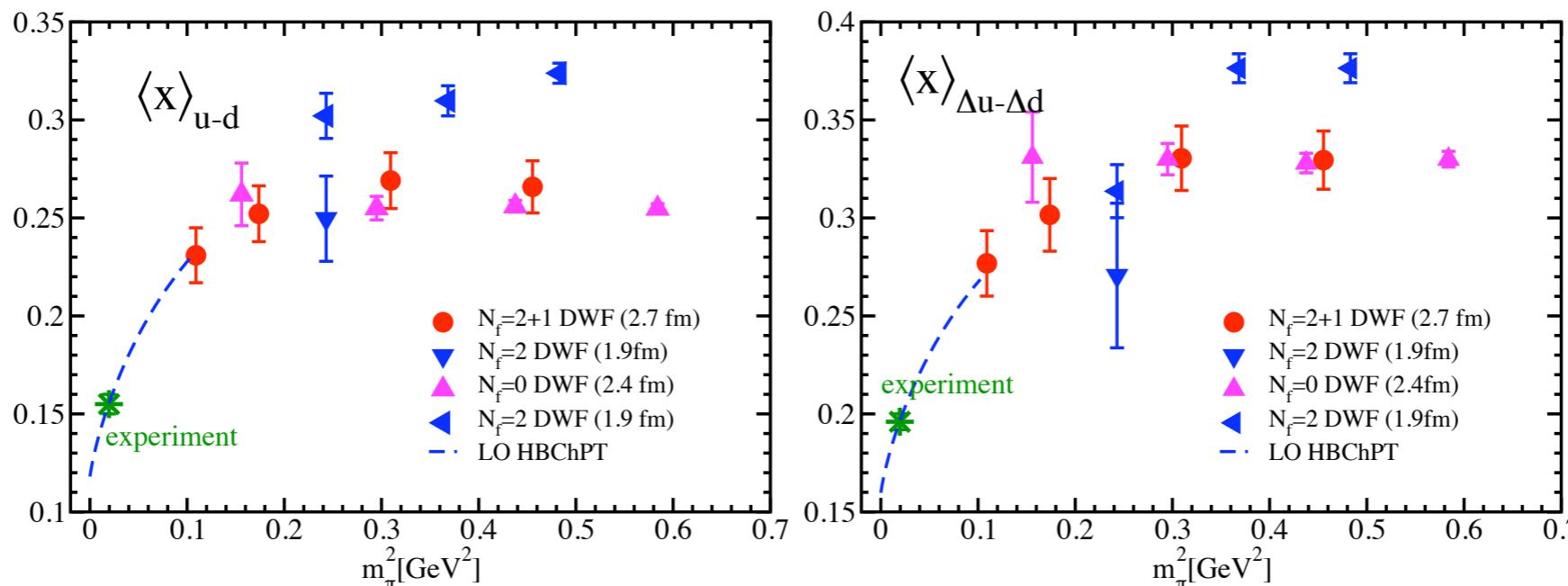
# Kozi Nakai (KEK)



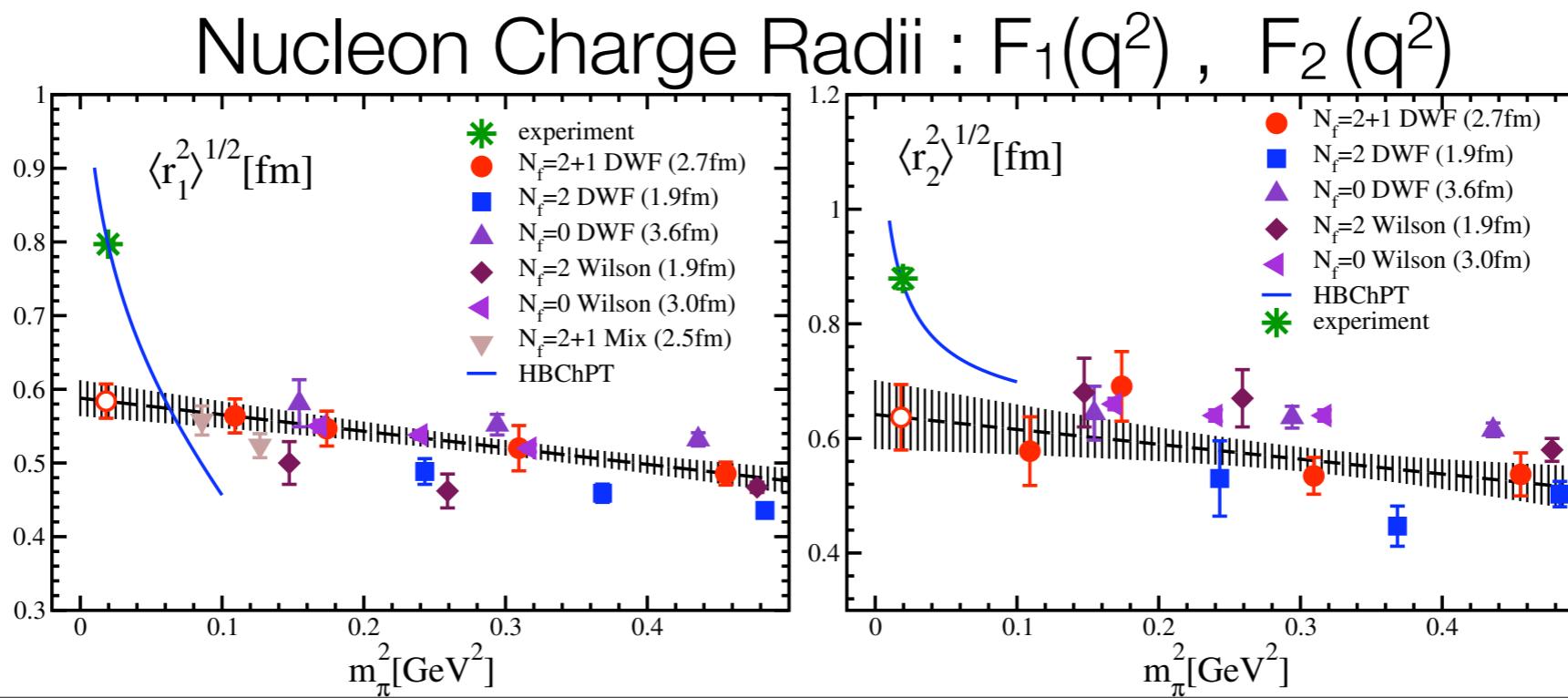
# The Structure of Hadrons : Cold Lattice QCD



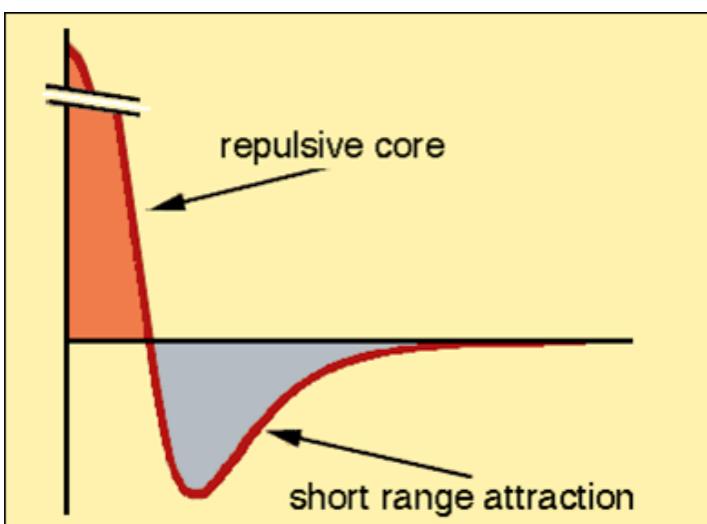
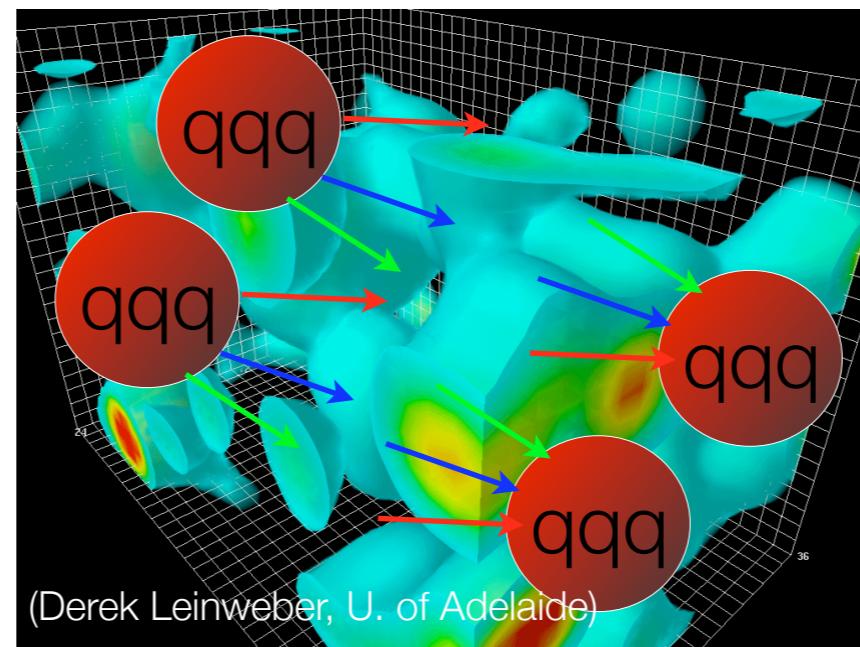
# Nucleon Parton Distributions and Charge Radii



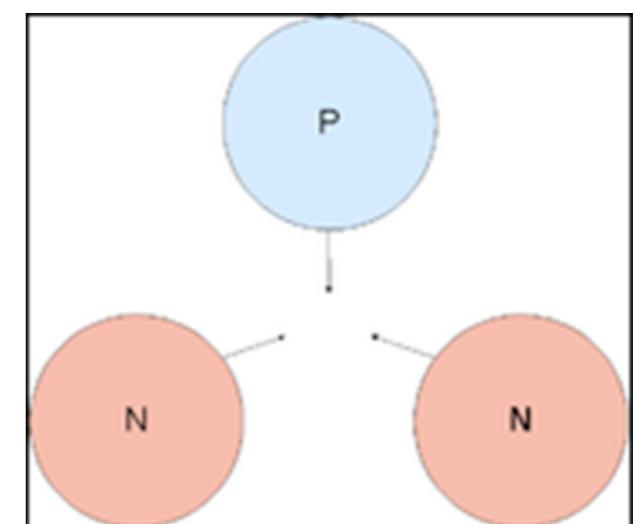
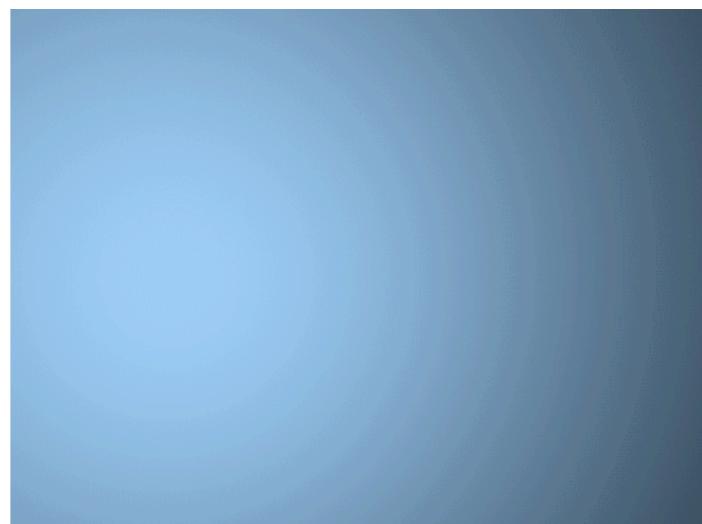
Iso-vector Quark Distributions : Momentum and Helicity



# Nuclear Forces and Multi-Hadron Systems



$\chi$ -symmetry



$\chi$ -symmetry



# NPLQCD



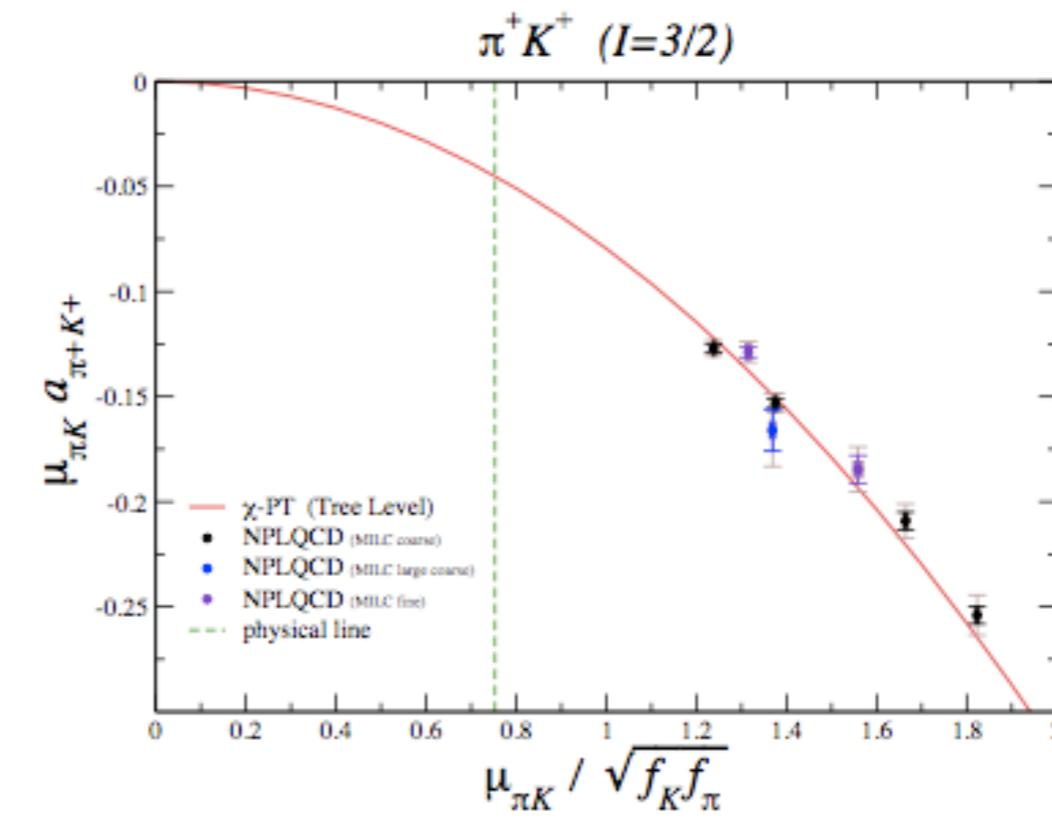
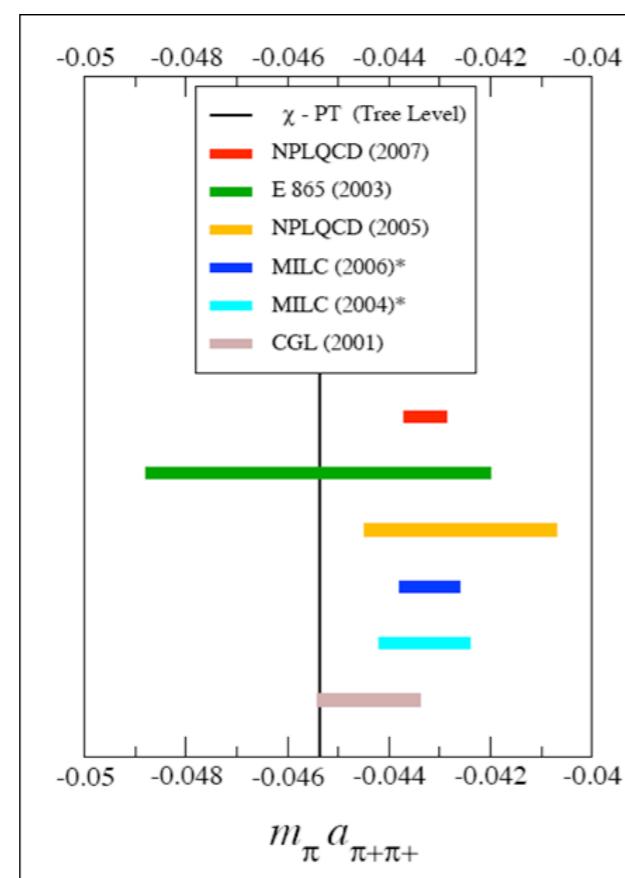
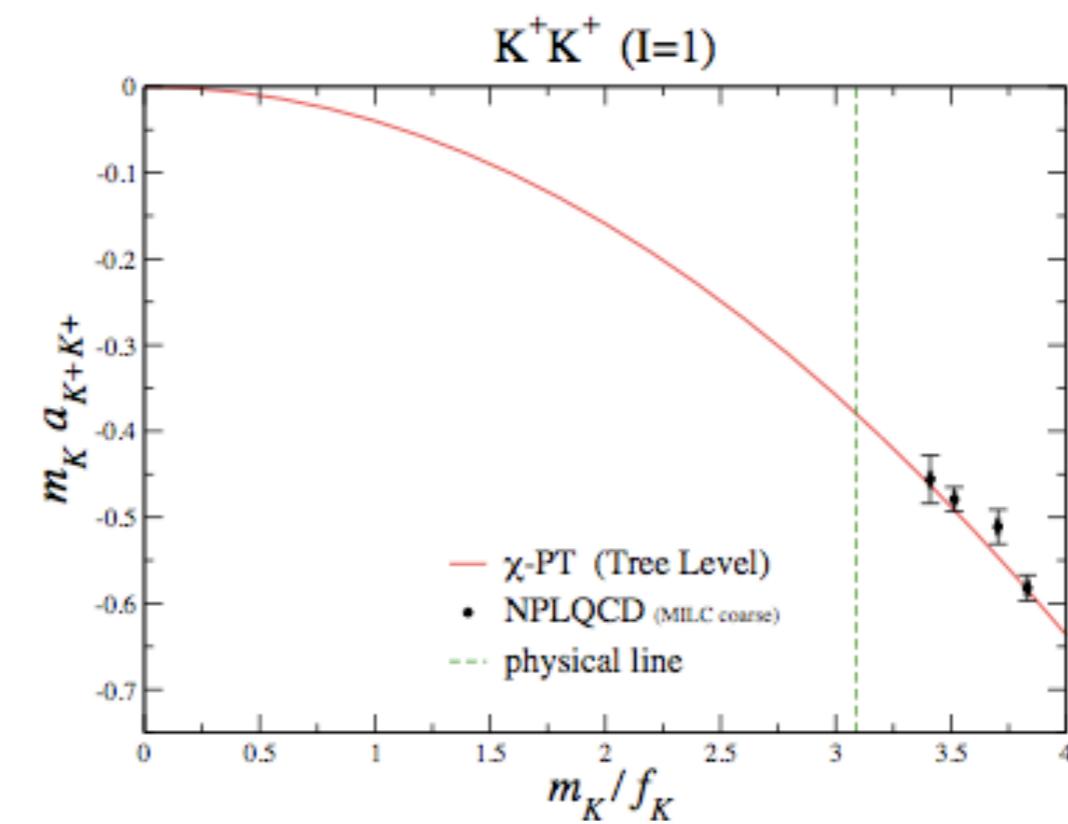
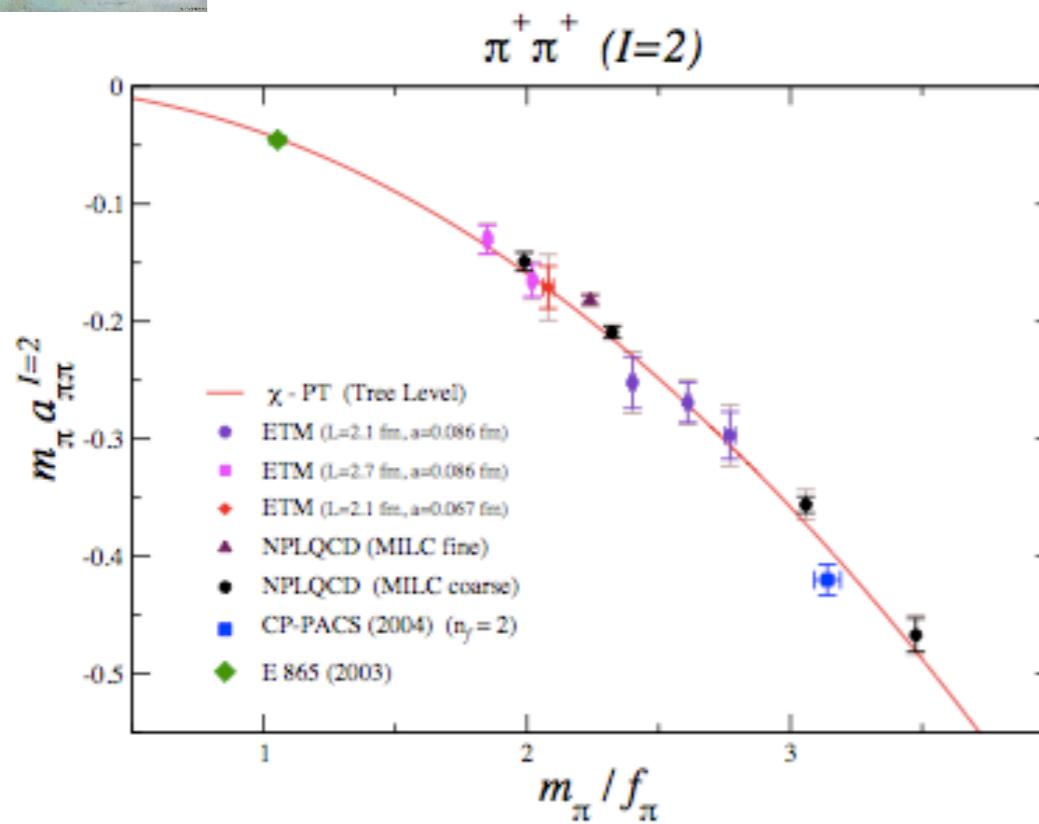
+



... to make predictions for the structure and interactions of nuclei using lattice QCD.

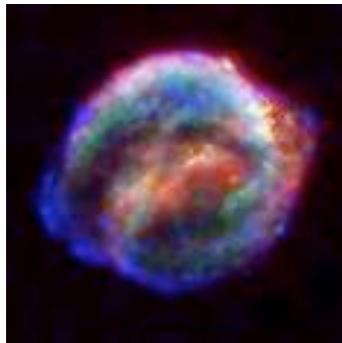


# Meson-Meson Scattering : A Puzzle!

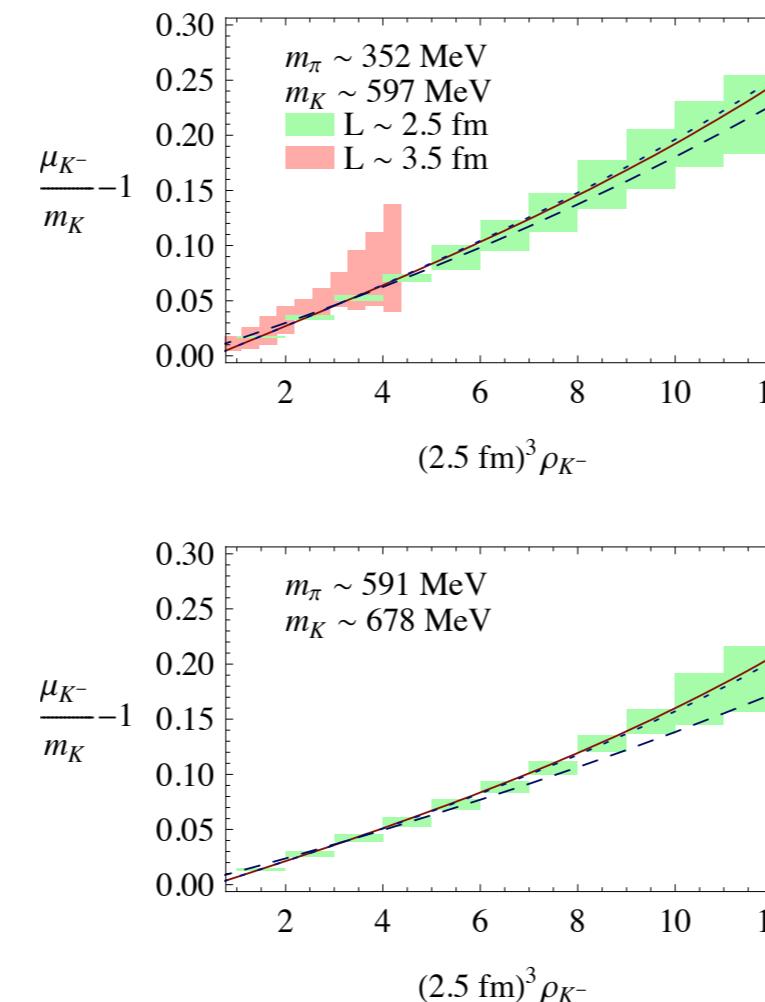
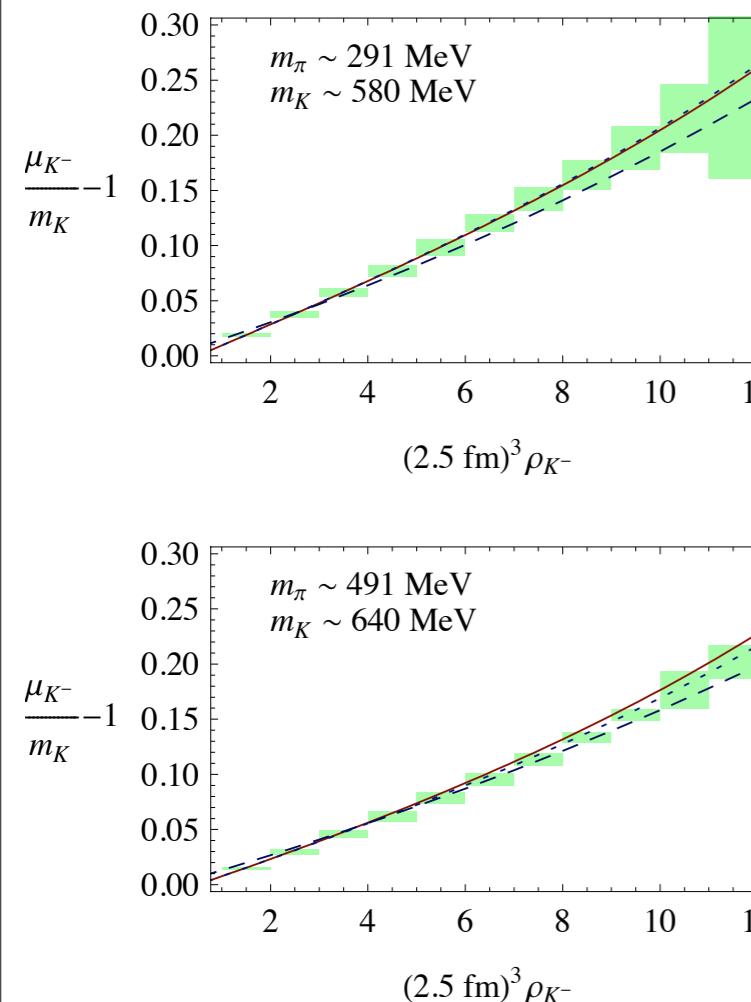




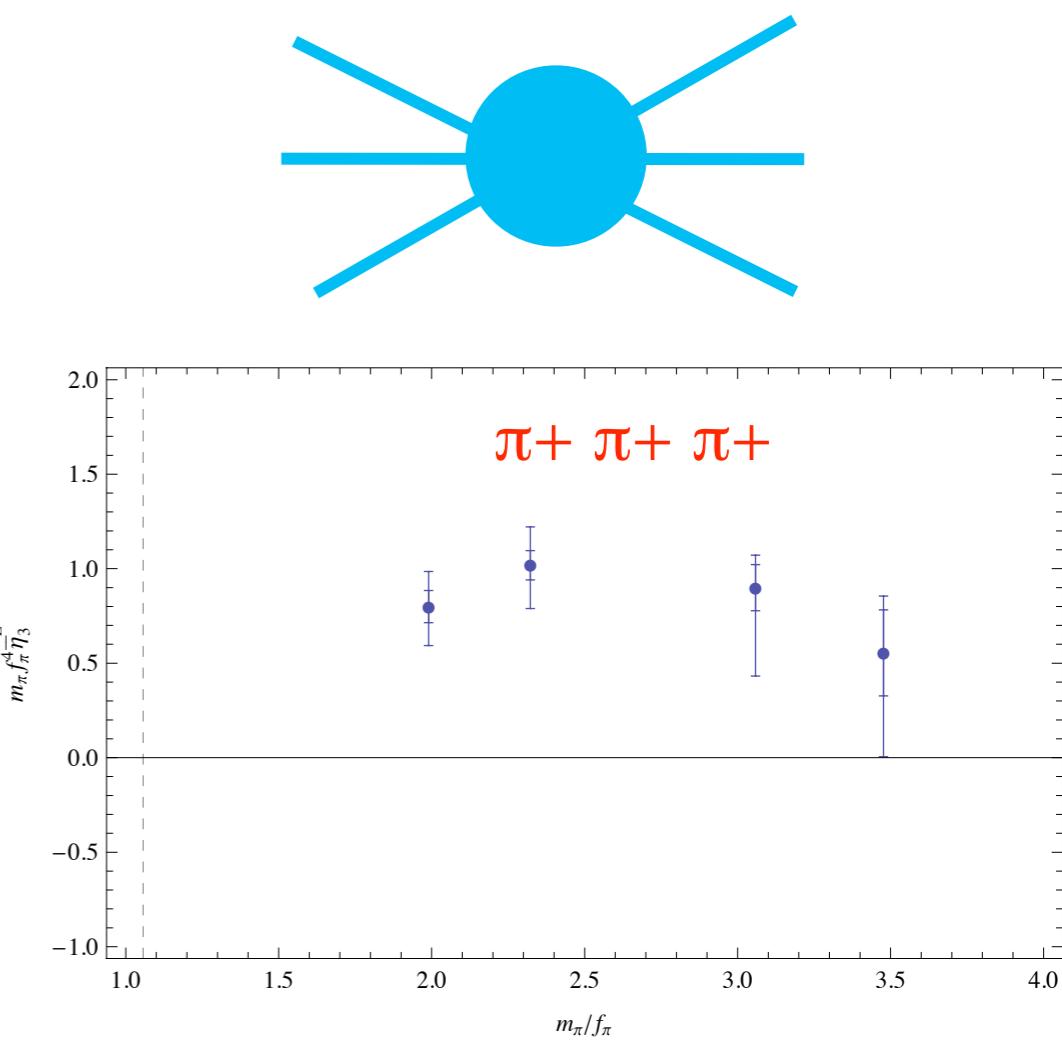
# Bose-Einstein Condensates of Mesons : Many-Body Physics



## Kaon Condensates



## Pion 3-Body Interaction

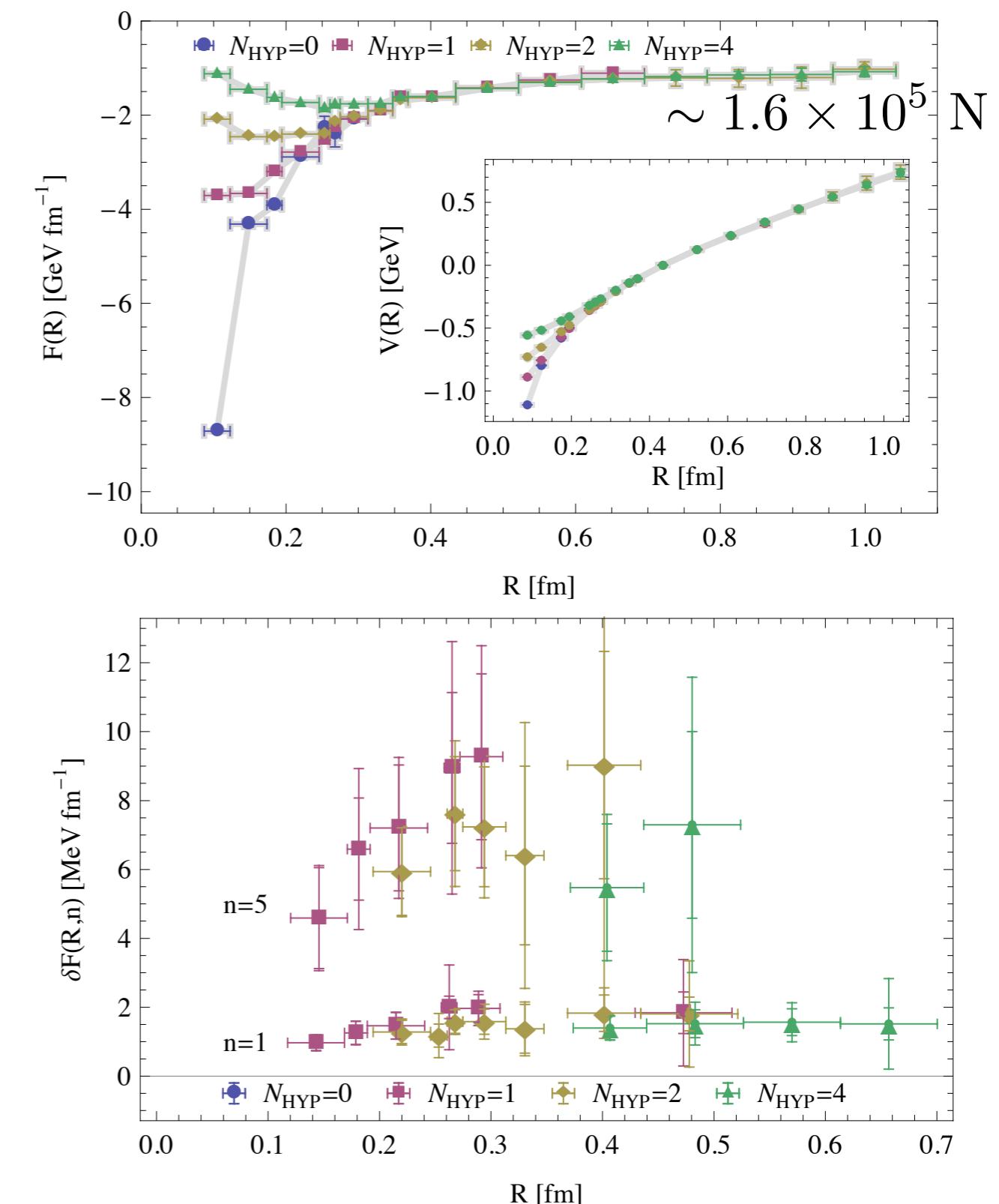
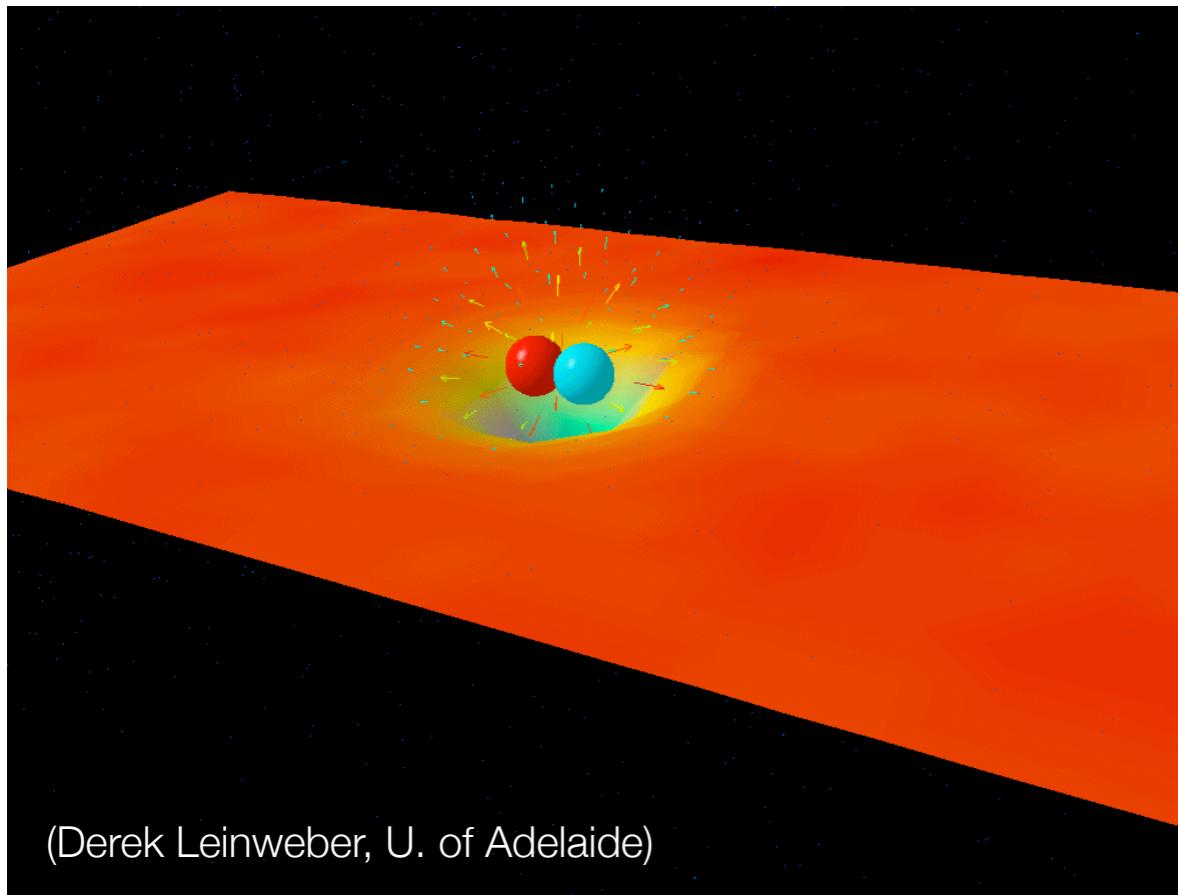


- Systems with kaons and pions
- Algorithm developed to include arbitrary numbers



# Color-Screening by Hadrons

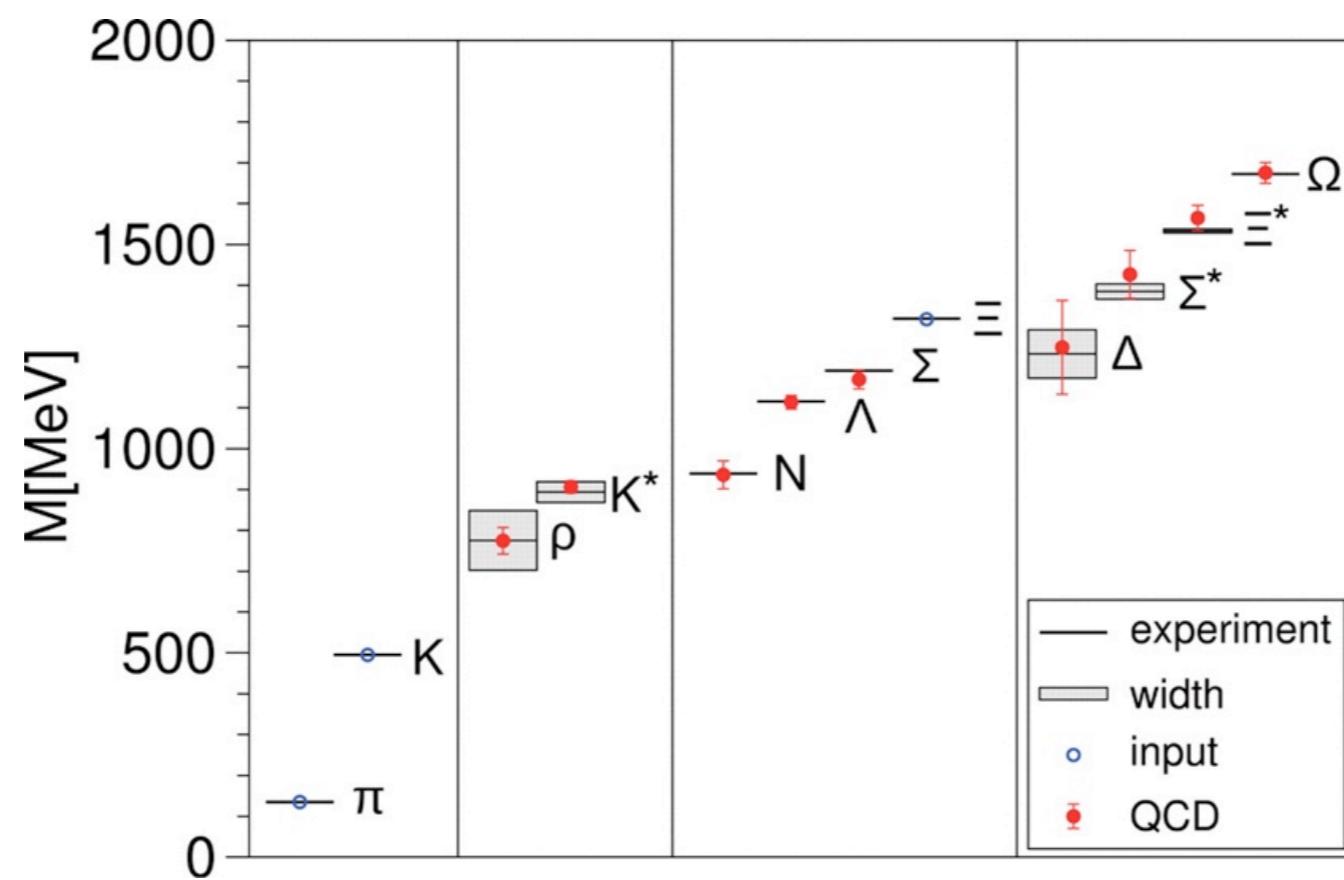
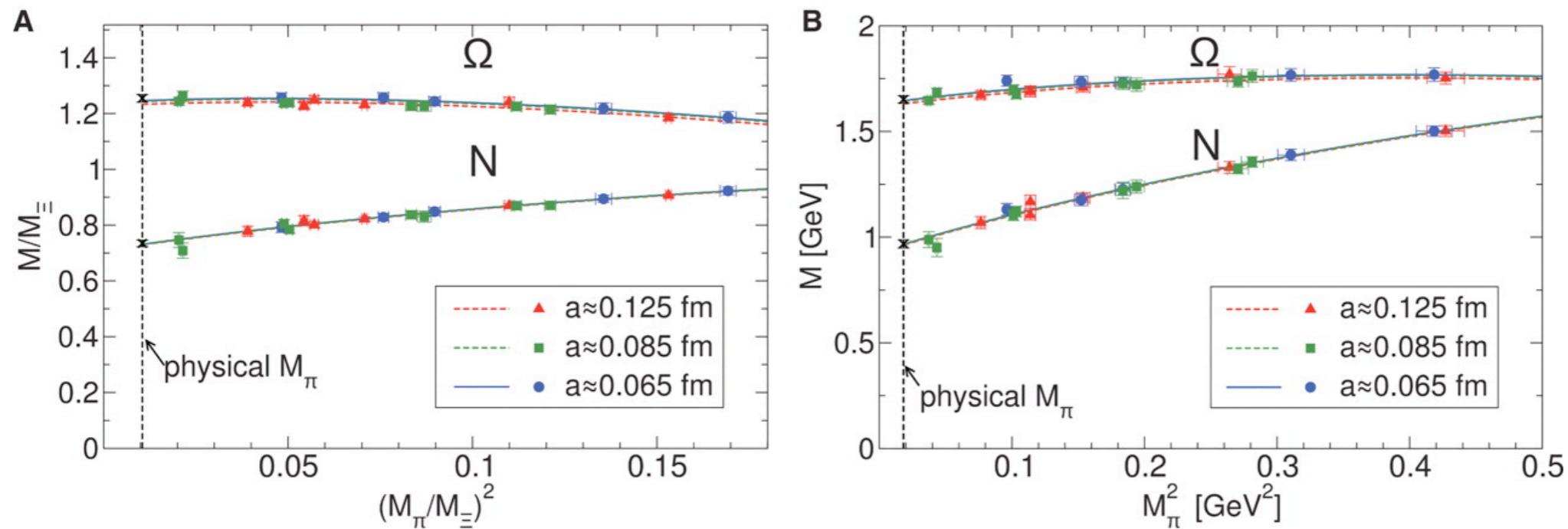
## Meson Condensate

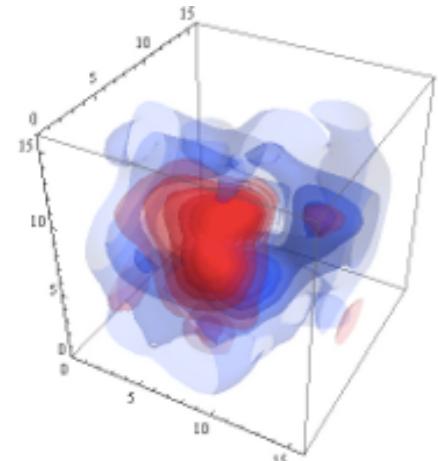




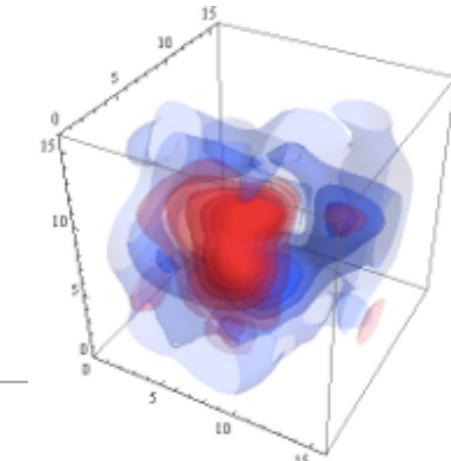
(2008)

# Near Physical Quark Masses



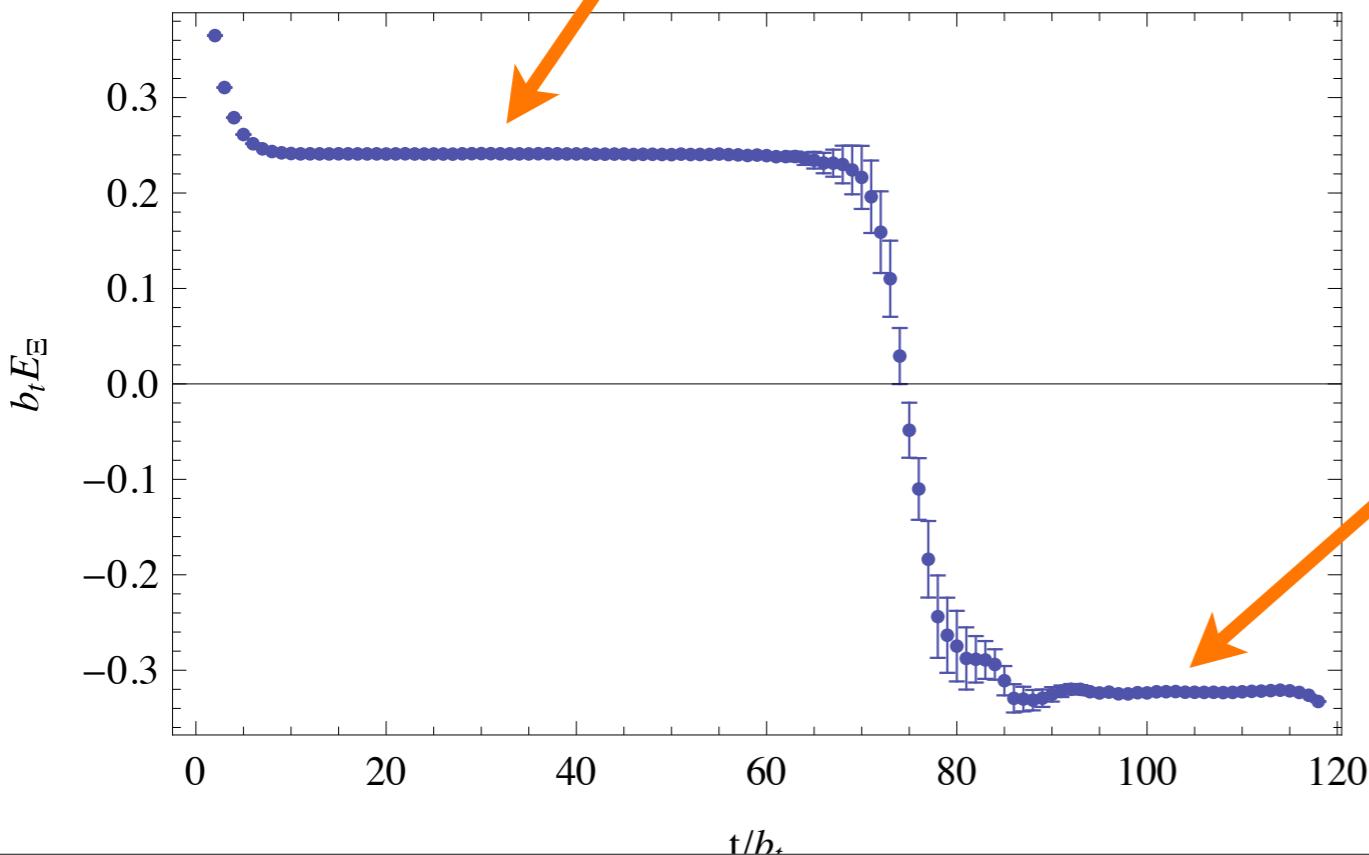


# Baryon Correlation Functions in Euclidean Space



$$C(t) = \int d^3\mathbf{x} \langle 0 | qqq(\mathbf{x}, t) \overline{qqq}(\mathbf{0}, 0) | 0 \rangle$$

$$C(t) = Z_n \exp^{-M_n t} + Z_{n\pi\pi} \exp^{-(M_n + 2M_\pi + \Delta E_{n\pi\pi})t} \\ + Z_{n\pi} \exp^{-(M_n + m_\pi + \Delta E_{n\pi})(T - t)} + \dots$$

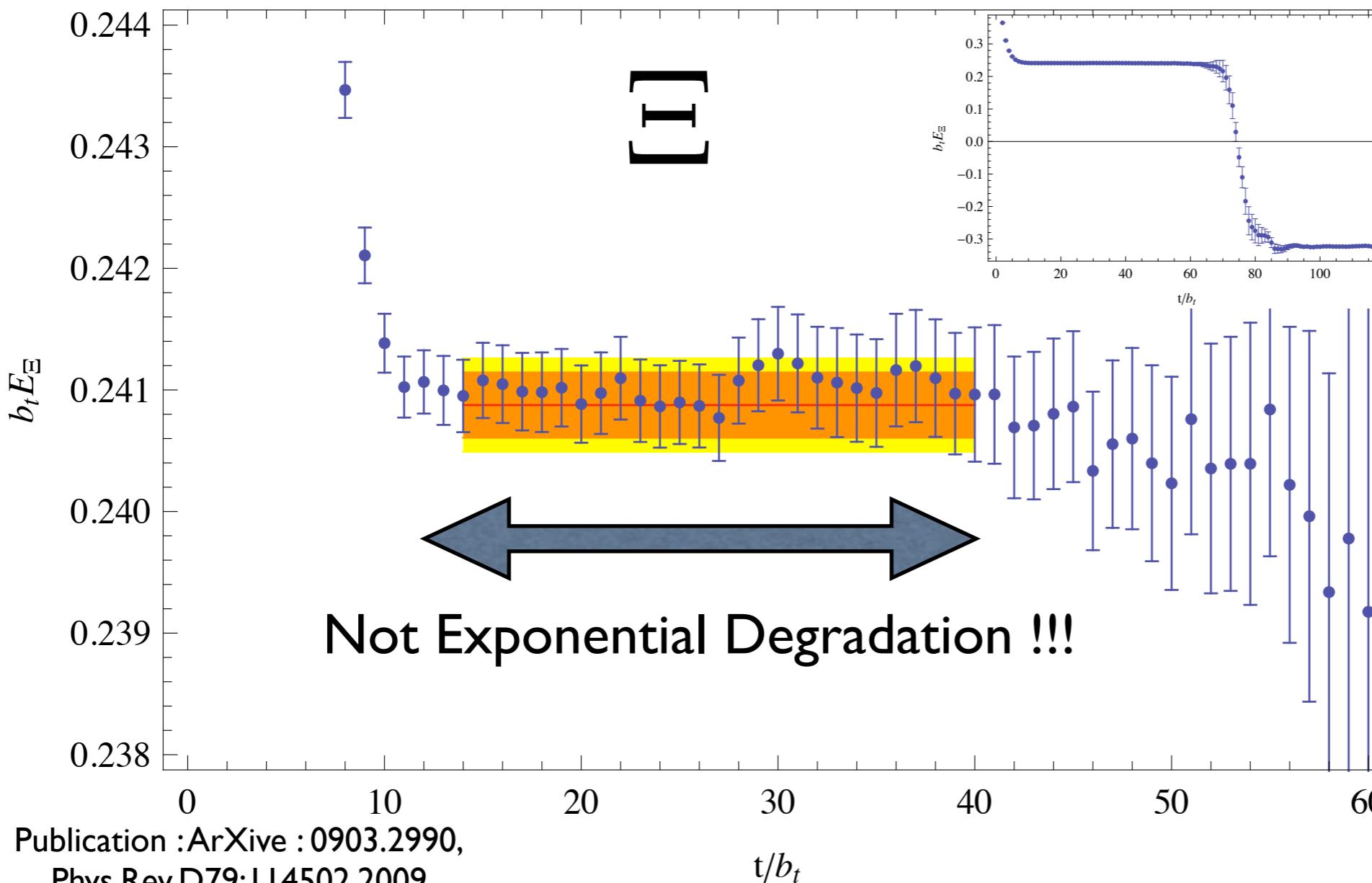


$$M_{\text{eff}}(t) = \frac{1}{t_J} \log \left( \frac{C(t)}{C(t + t_J)} \right)$$



# High Statistics Calculations on the Jefferson Lab Anisotropic Clover Lattices : Single Baryon Systems

$b \sim 0.123$  fm  
 $20 \times 20 \times 20 \times 128$   
pion  $\sim 390$  MeV

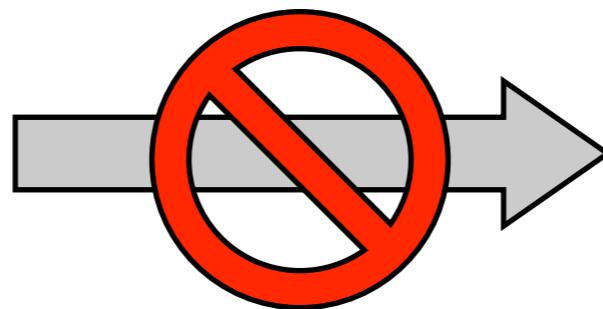


# Maiani-Testa Theorem

## Implications for Nuclear Physics !

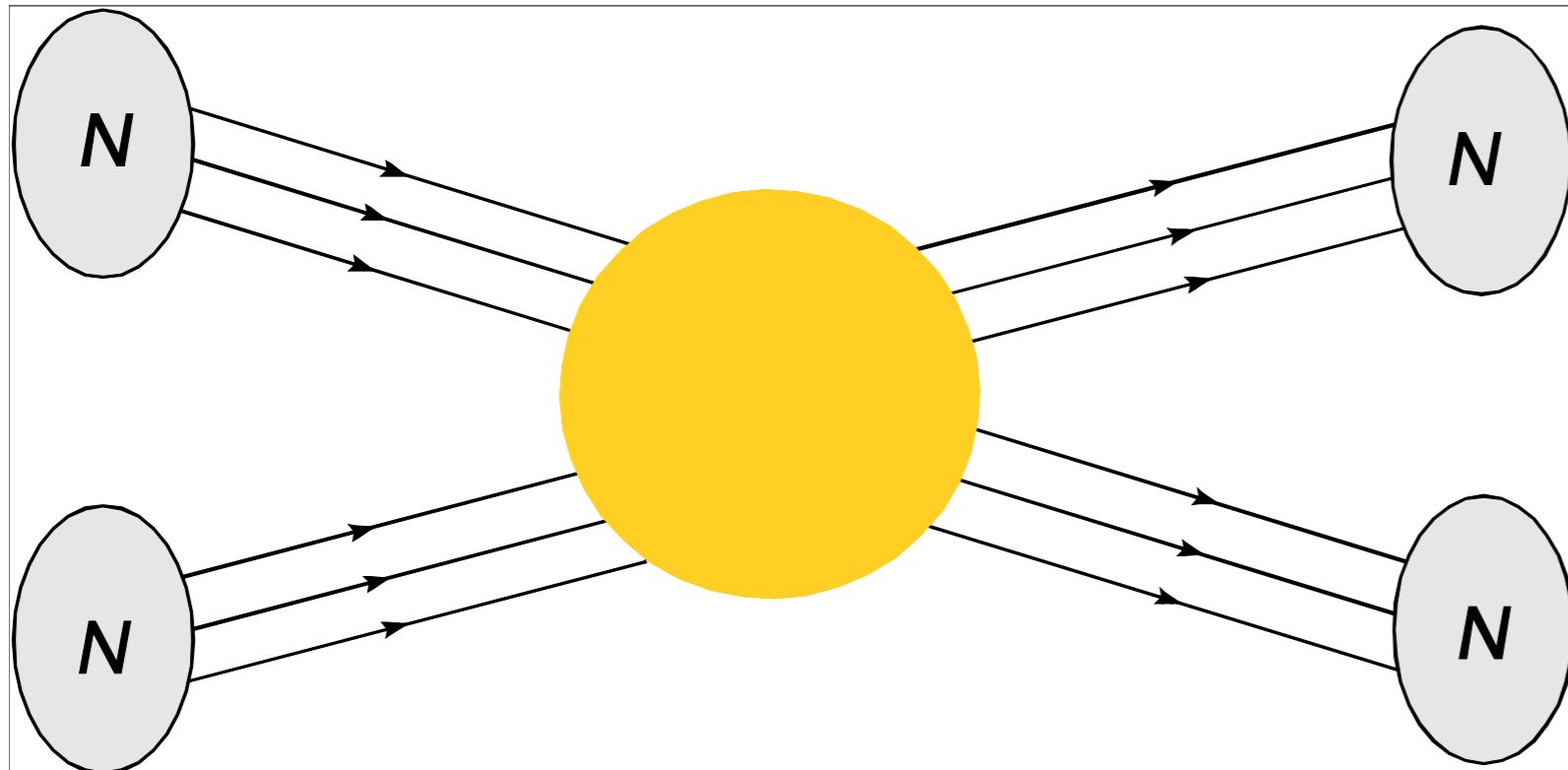
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Away from Kinematic Thresholds



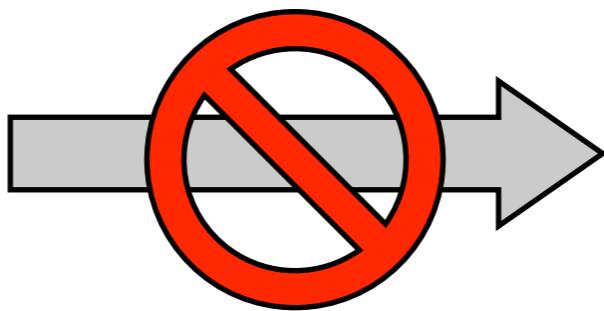
# Maiani-Testa Theorem

## Implications for Nuclear Physics !



Away from Kinematic Thresholds

$G_{NN}(s)^{\text{Euclidean}}$

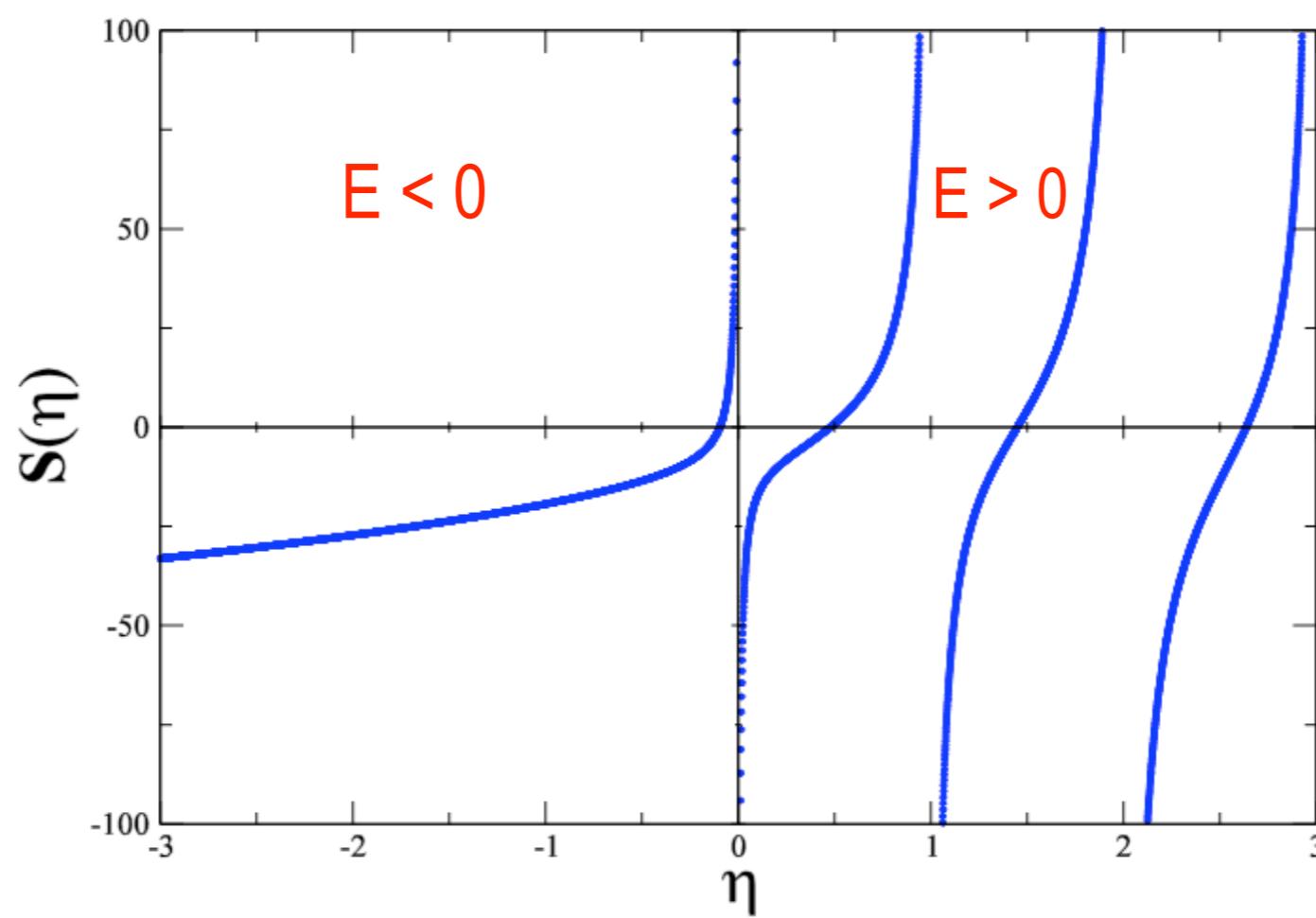
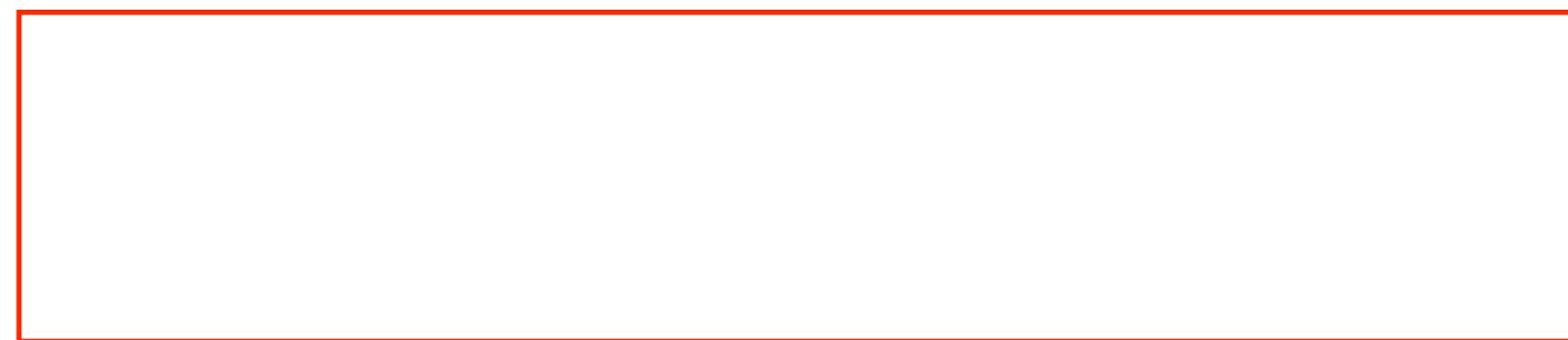


$G_{NN}(s)^{\text{Minkowski}}$

# Energy Eigenvalues and the Luscher Relation

Below Inelastic Thresholds :  
Measure on lattice

$$\delta E = 2\sqrt{p^2 + m^2} - 2m$$

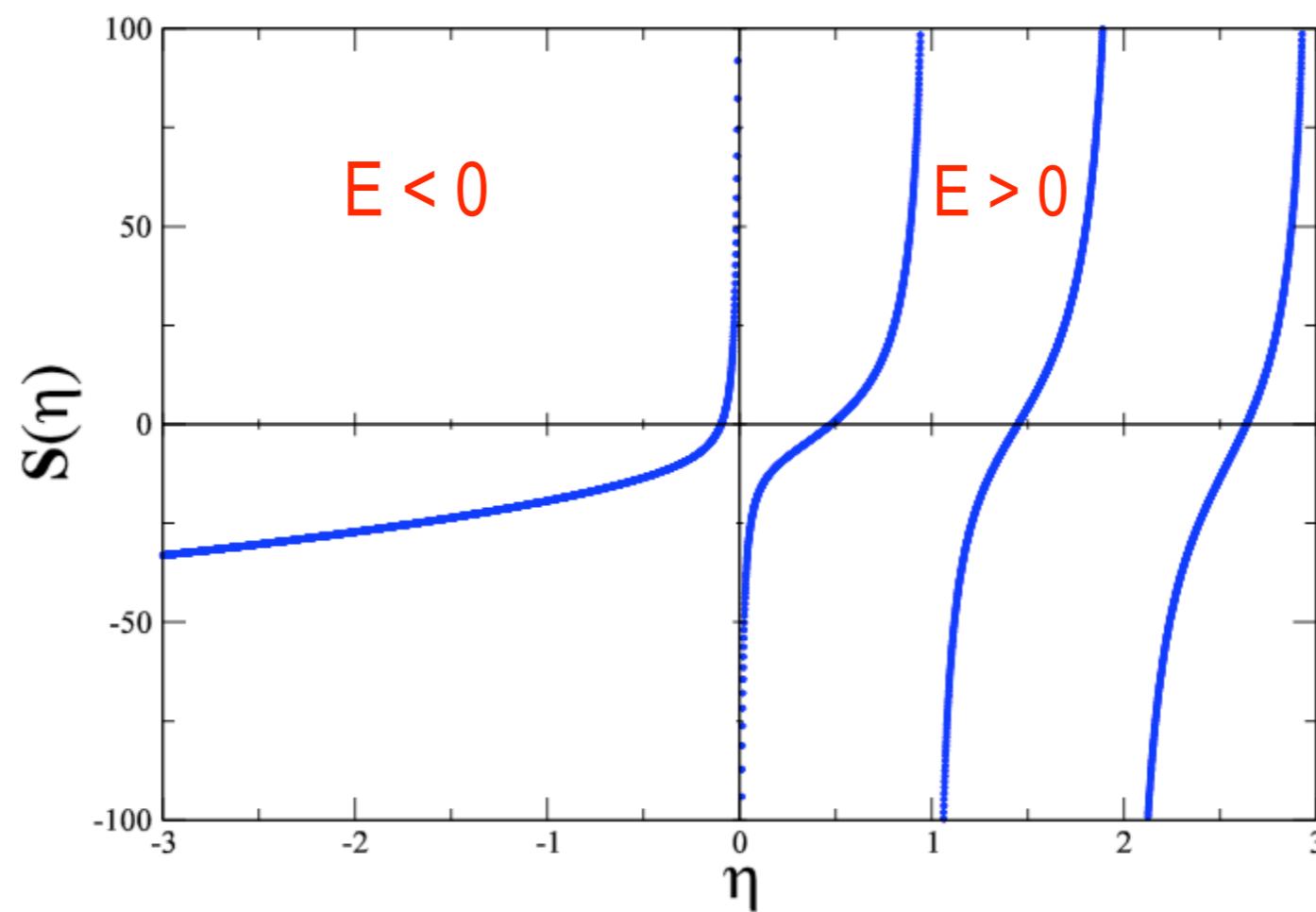


# Energy Eigenvalues and the Luscher Relation

Below Inelastic Thresholds :  
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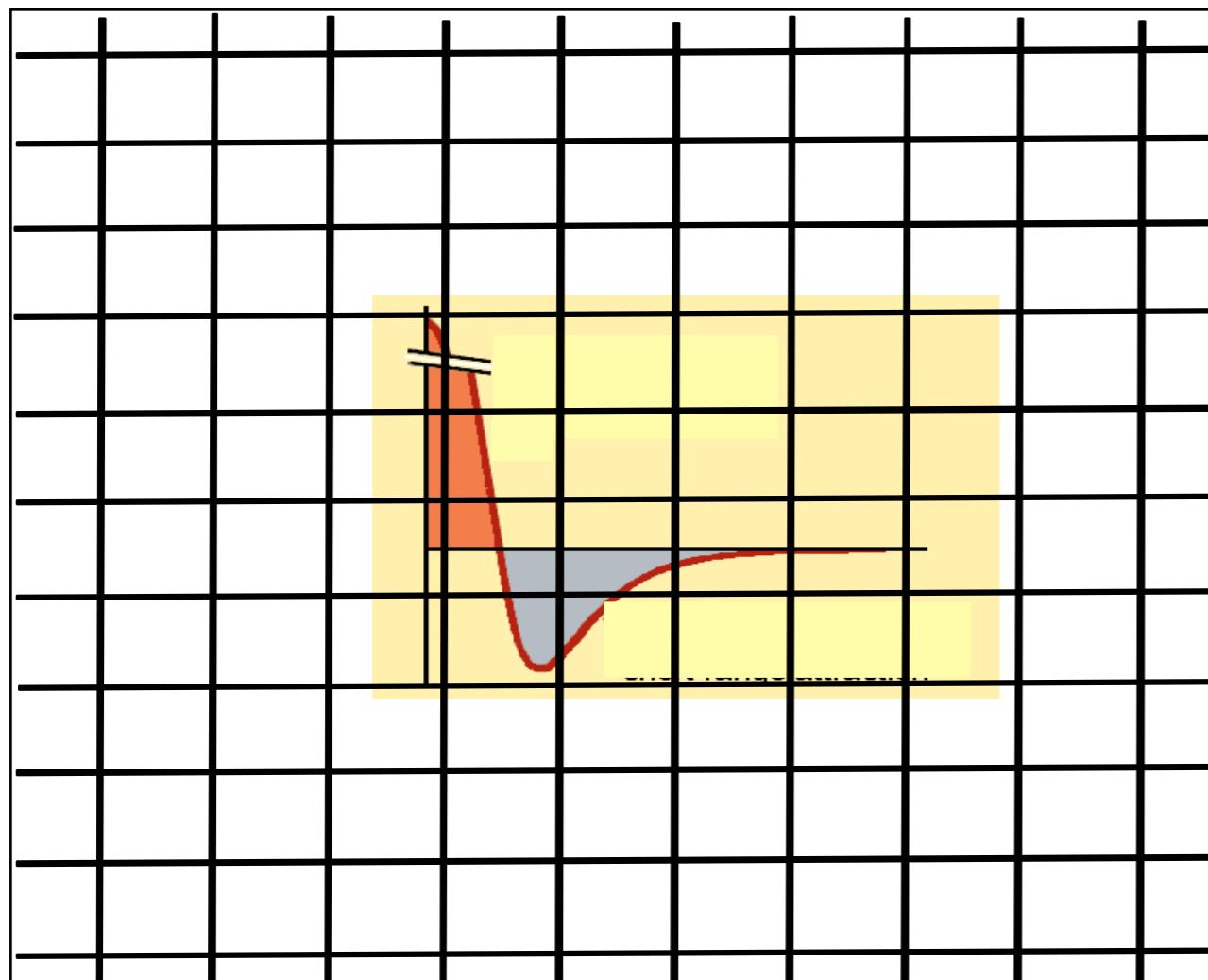
$$\delta E = 2\sqrt{p^2 + m^2} - 2m$$

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S} \left( \left( \frac{Lp}{2\pi} \right)^2 \right)$$



# Large Scattering Lengths are OK !

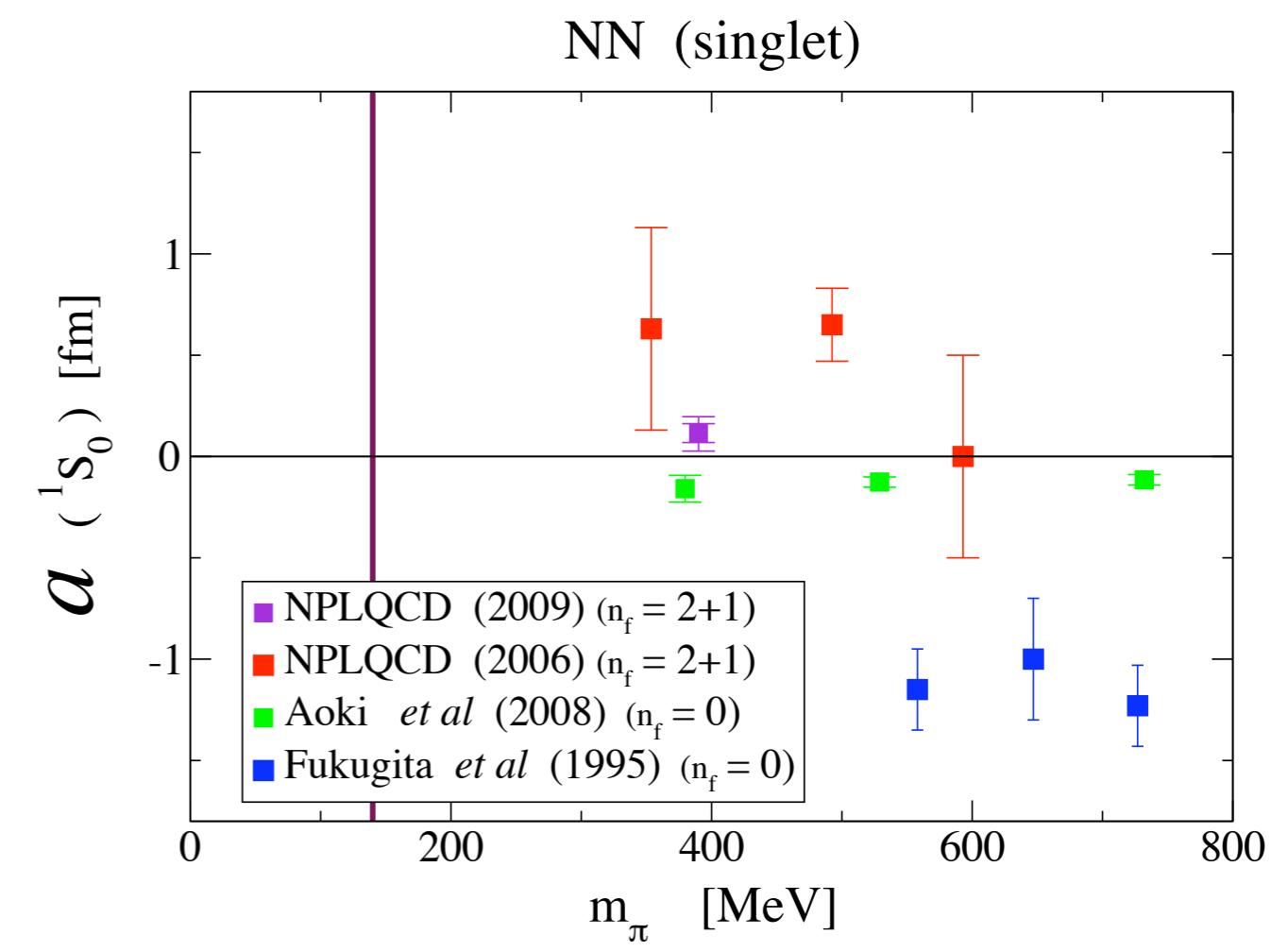
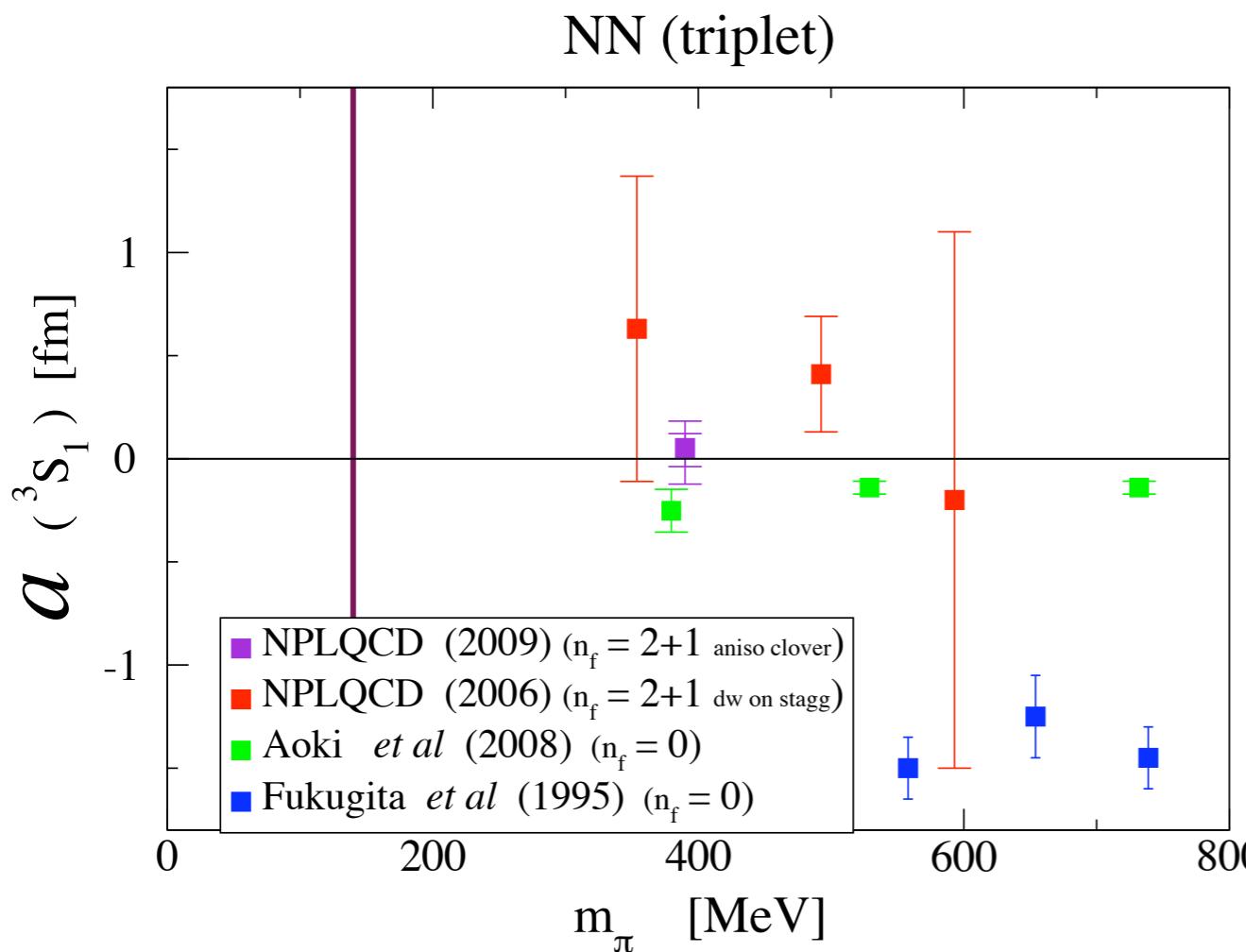
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Require :  $L \gg r_0$   
but ANY a



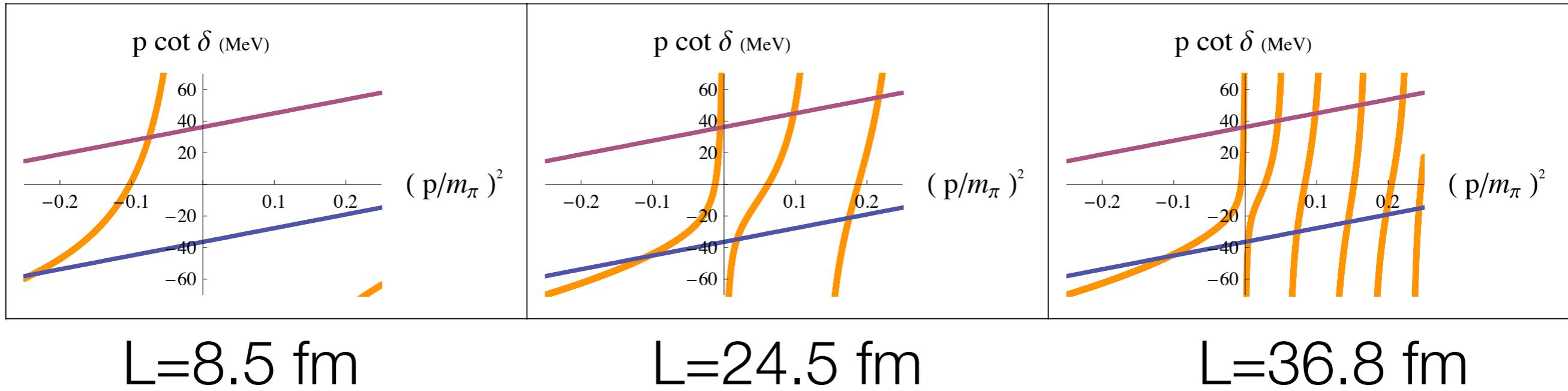
# NN Scattering



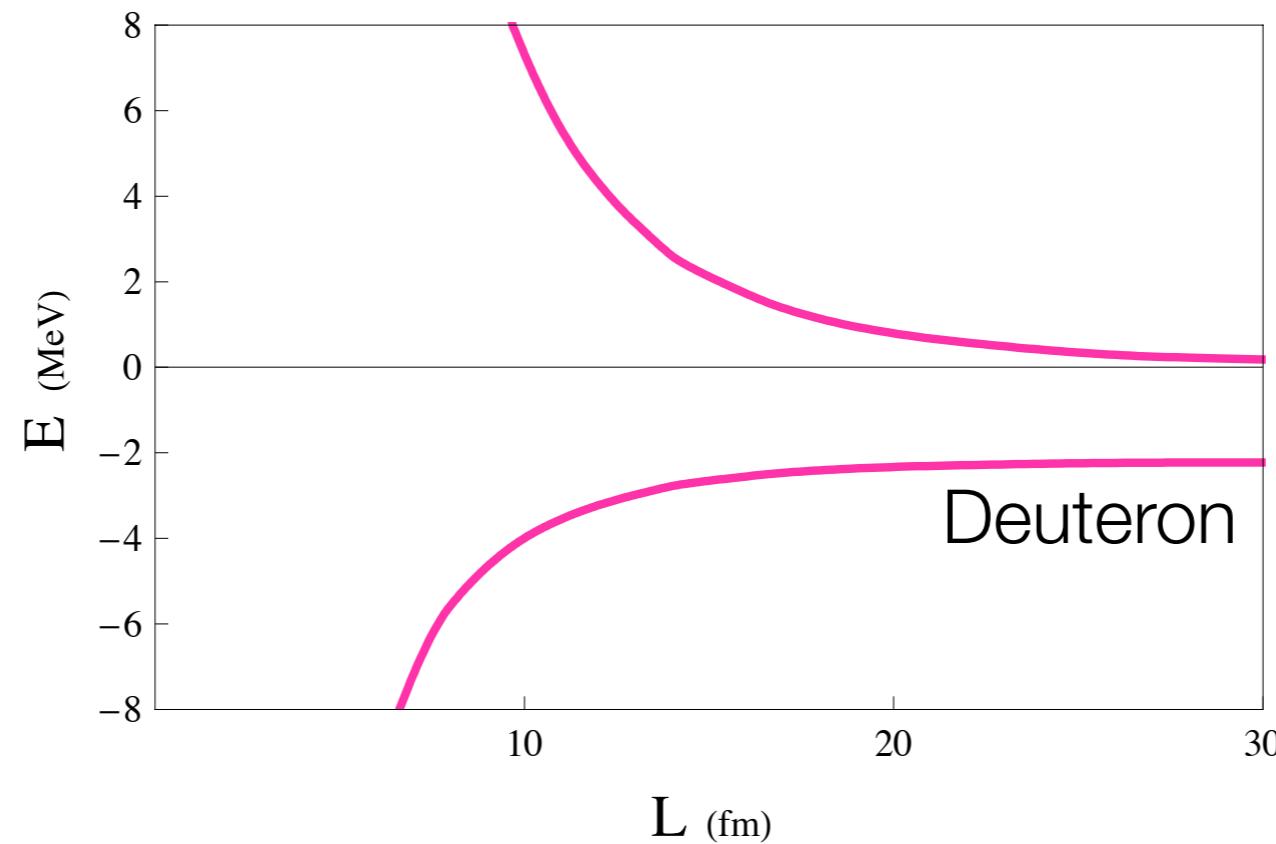
${}^3S_1$ - ${}^3D_1$  : pn : deuteron

${}^1S_0$  : pp , pn , nn

# Expectations at Physical Quark Masses : $^3S_1$ - $^3D_1$



Need pions in EFT !



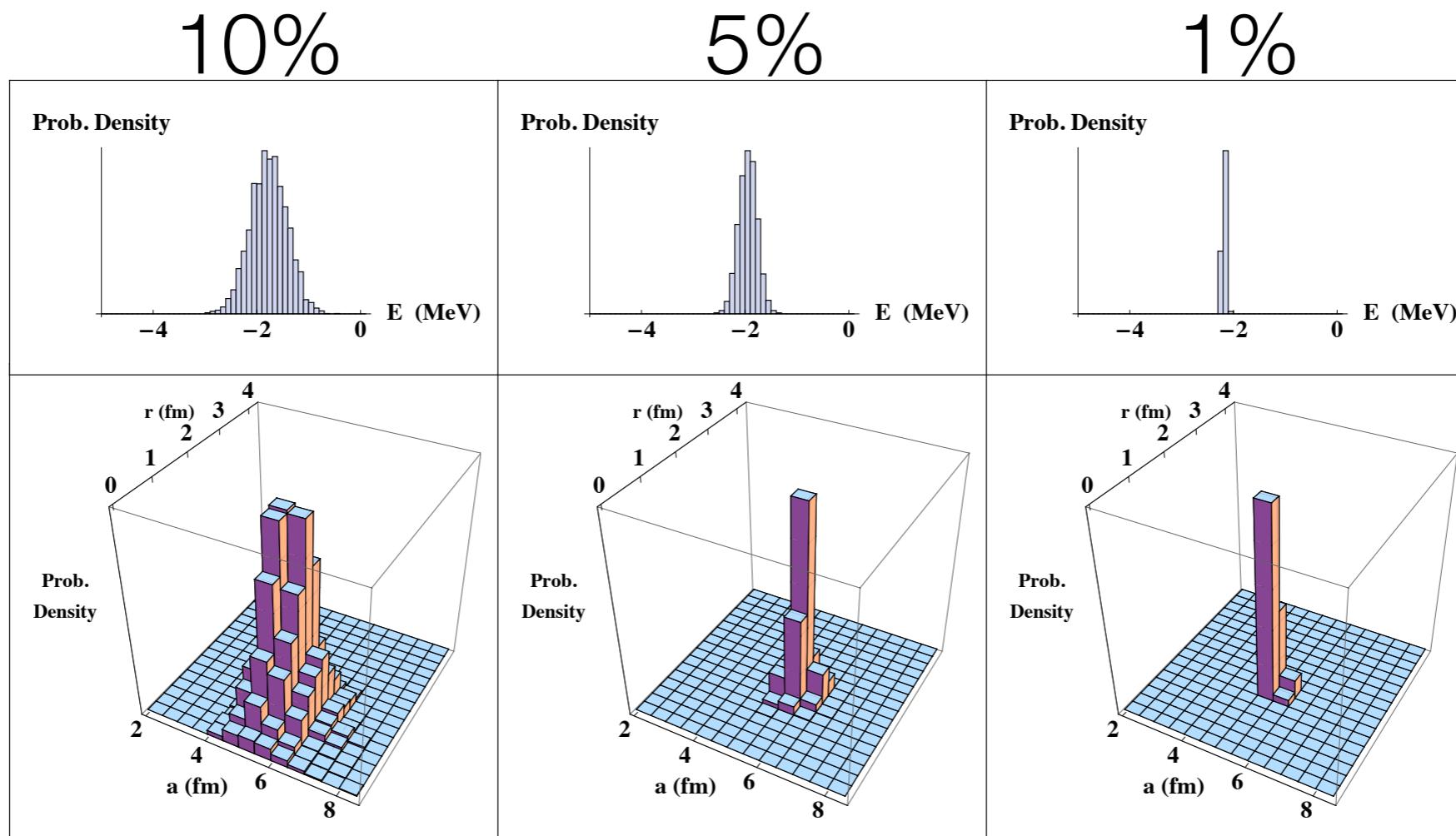
# Simulated Calculations of the Deuteron

(NOT actual calculations)

Precision Level of Energy Shift	Bound State Energy (MeV)	1 <sup>st</sup> ContinuumLevel (MeV)
0%	-3.147	4.005
1%	-3.111 ± 0.031	4.015 ± 0.040
5%	-2.95 ± 0.16	4.24 ± 0.20
10%	-2.66 ± 0.31	3.65 ± 0.40

$$E_D \sim 2 \text{ GeV}$$

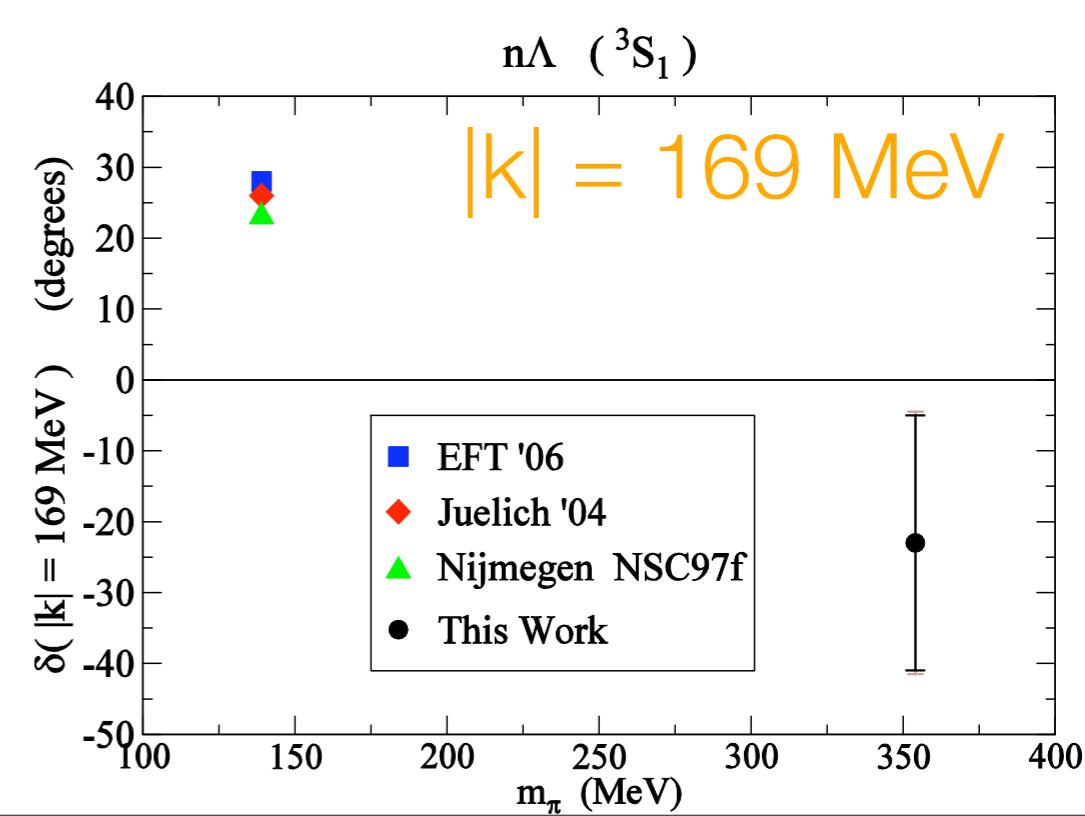
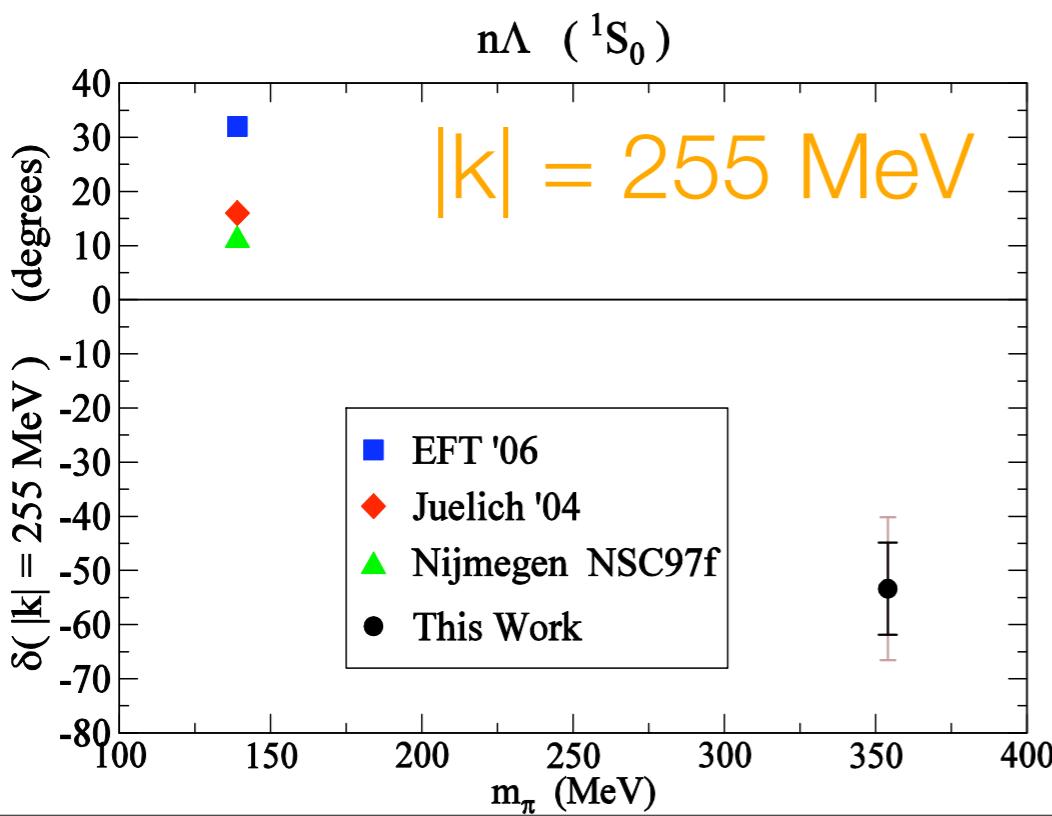
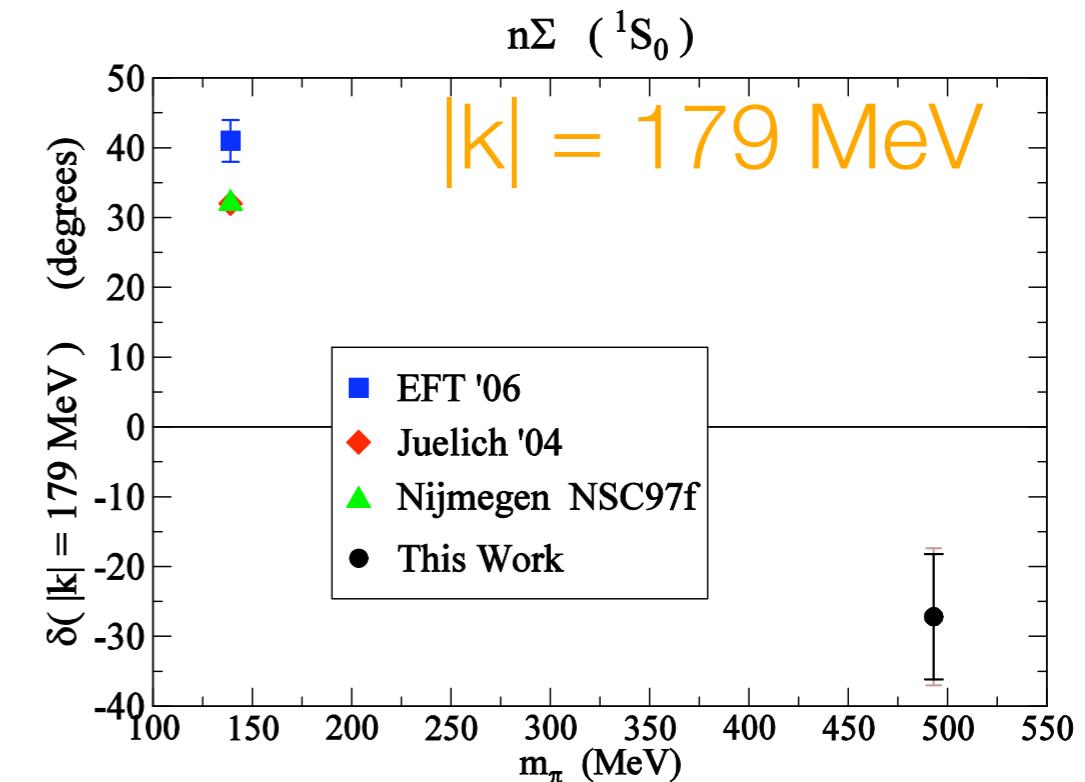
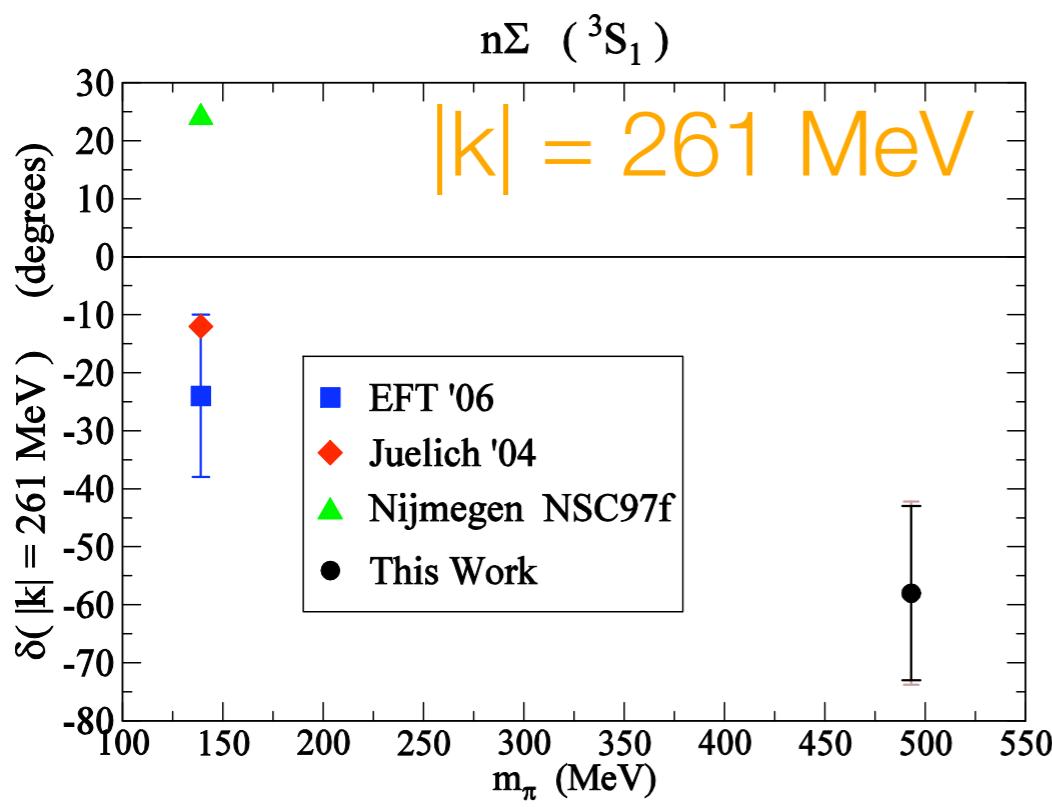
$$\Delta E_D \sim 2 \text{ MeV}$$





# Hyperon-N Interactions

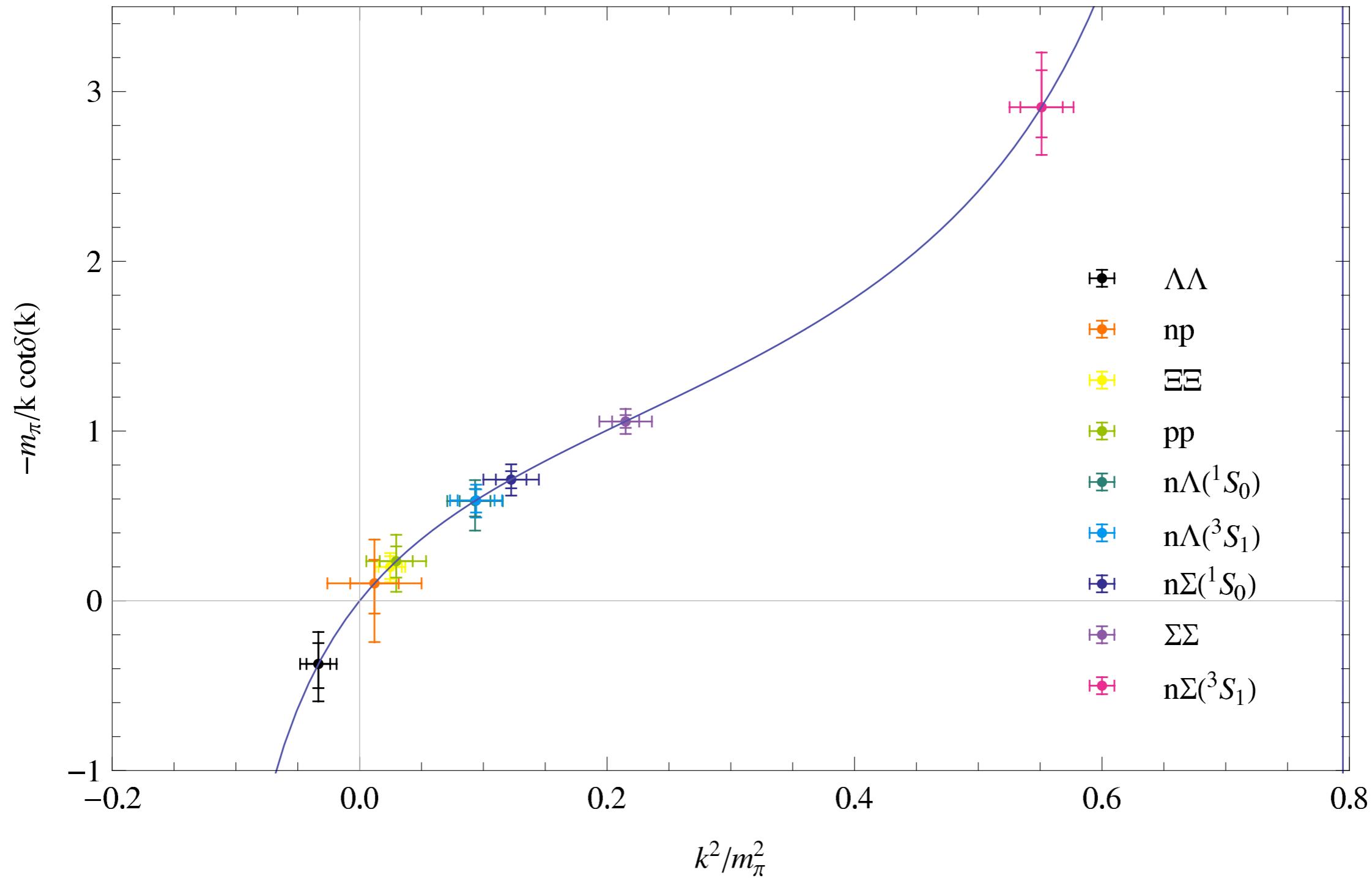
Nucl.Phys.A794:62-72,2007.  
e-Print: hep-lat/0612026





# Baryon-Baryon Scattering

$20^3 \times 128$  anisotropic lattices,  $m_\pi \sim 390$  MeV



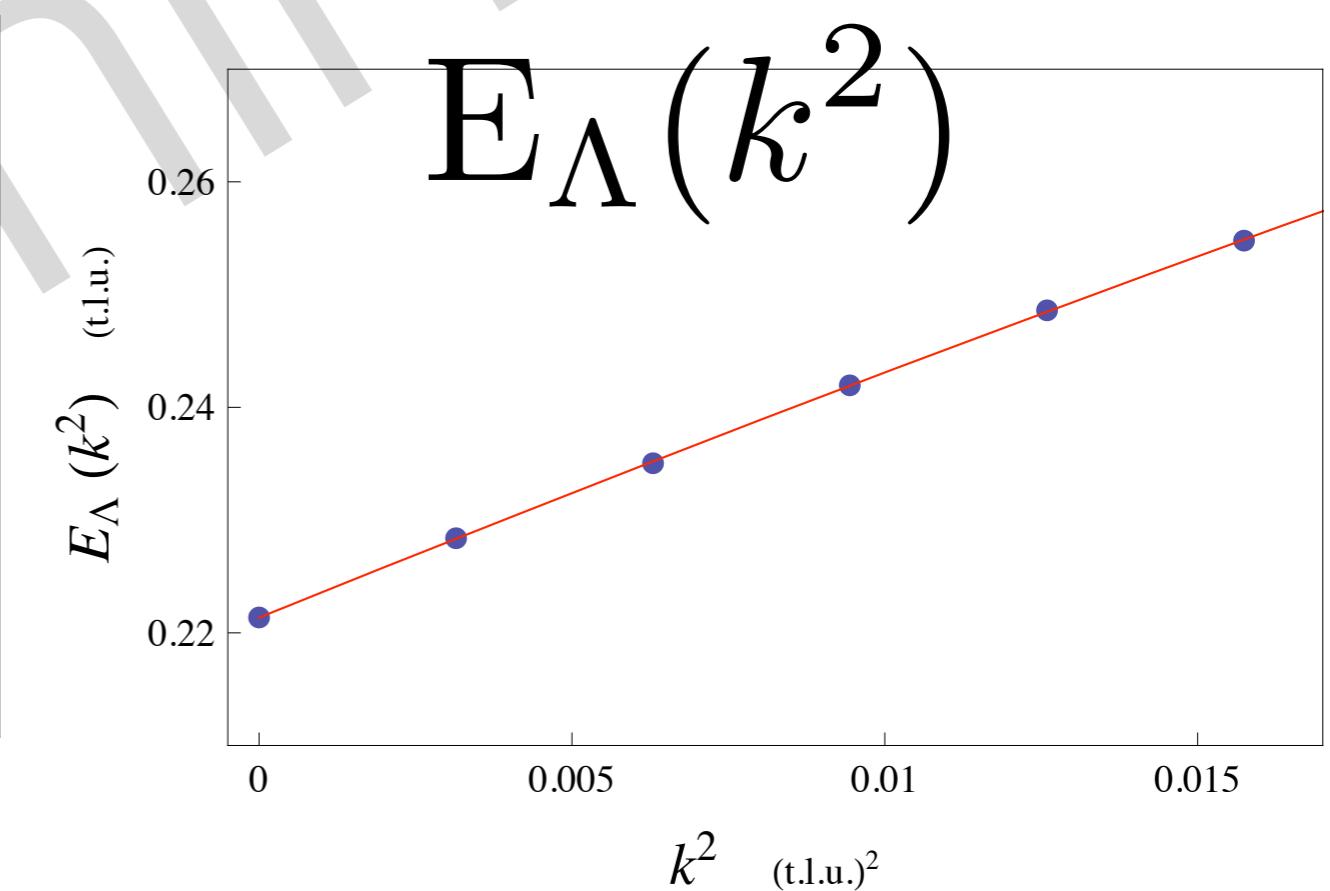
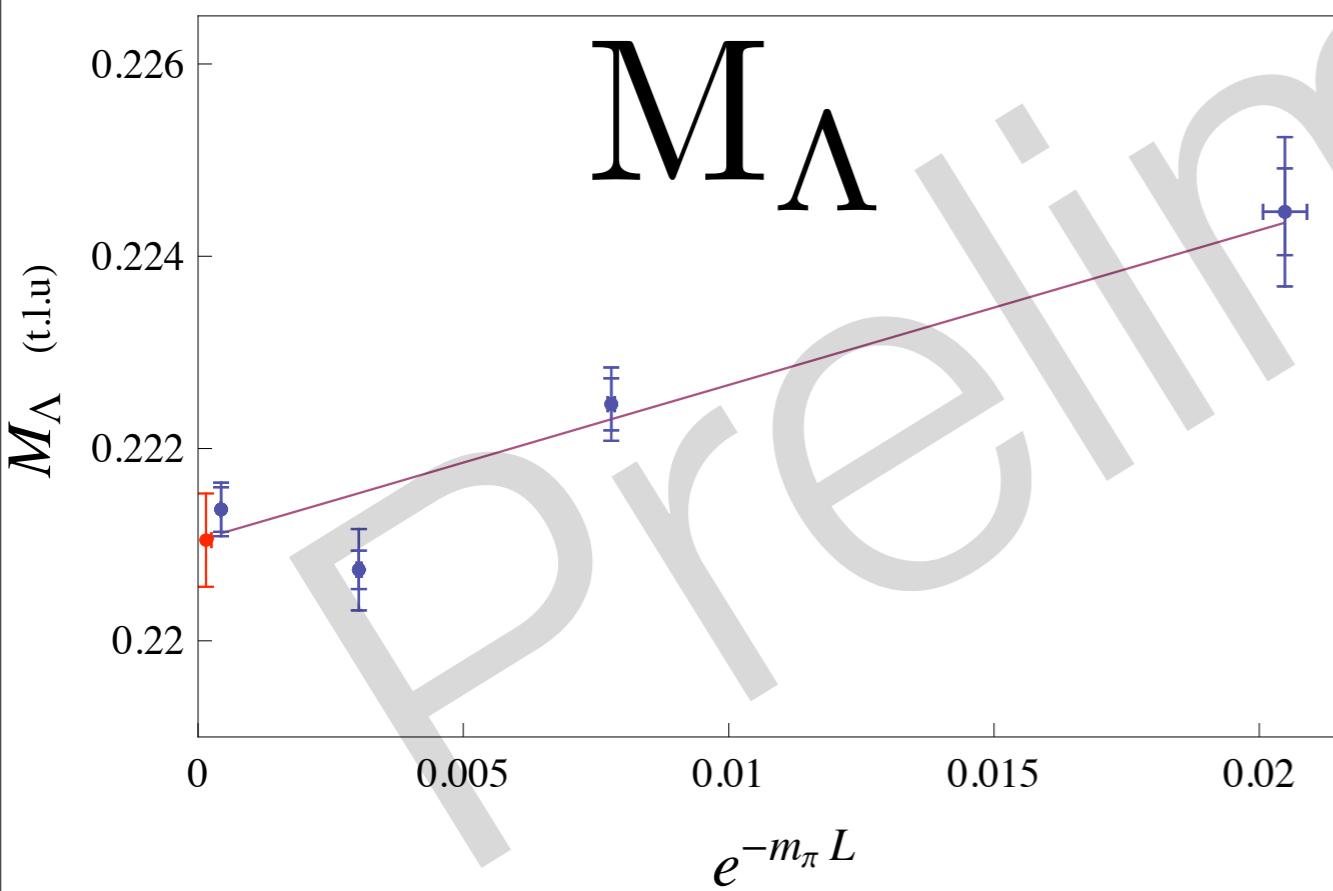


# Current Status in 4 Volumes

$m_\pi \sim 390$  MeV

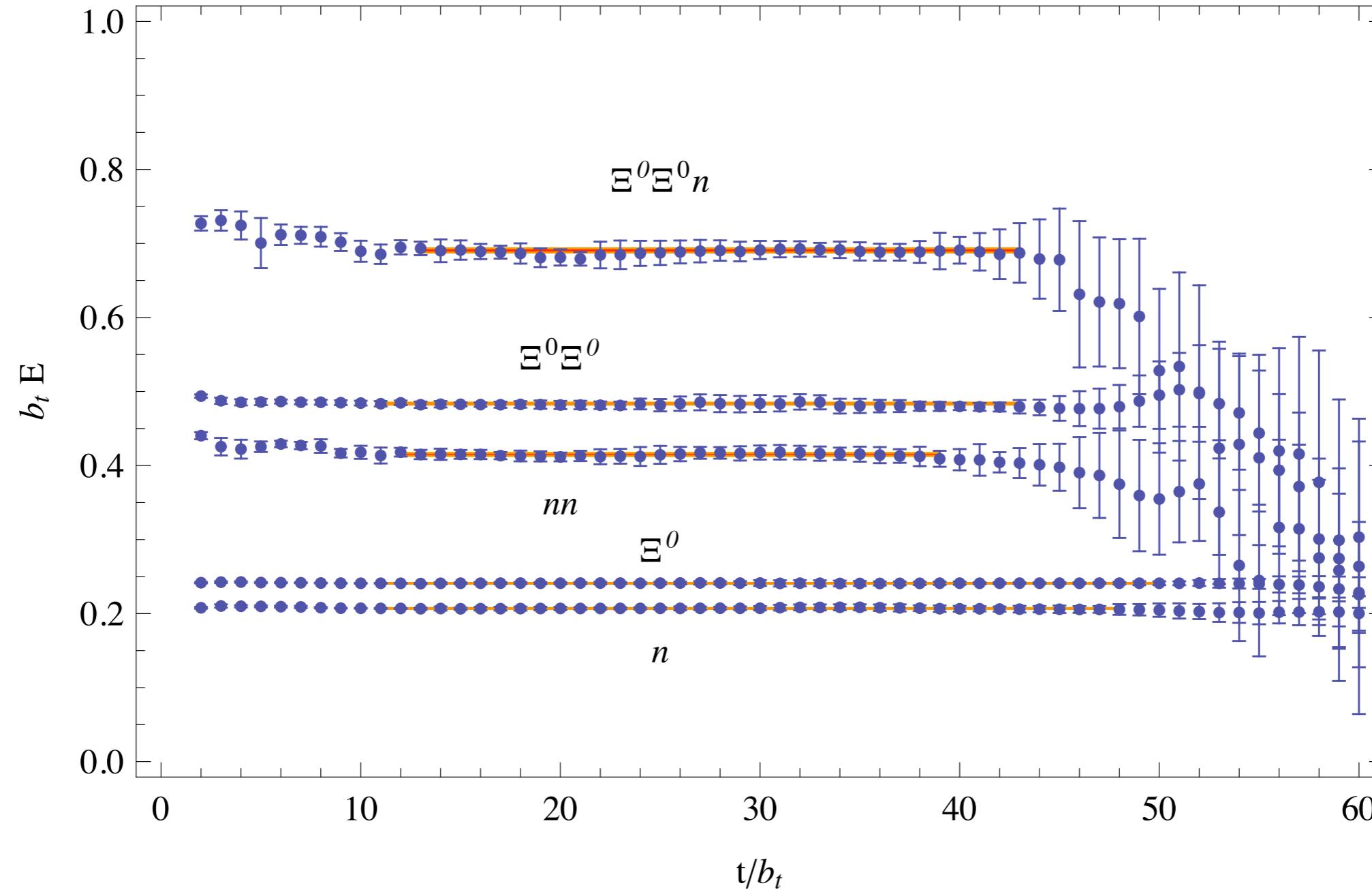
Lattice Volume  
Dependence

Energy-Momentum  
Relation



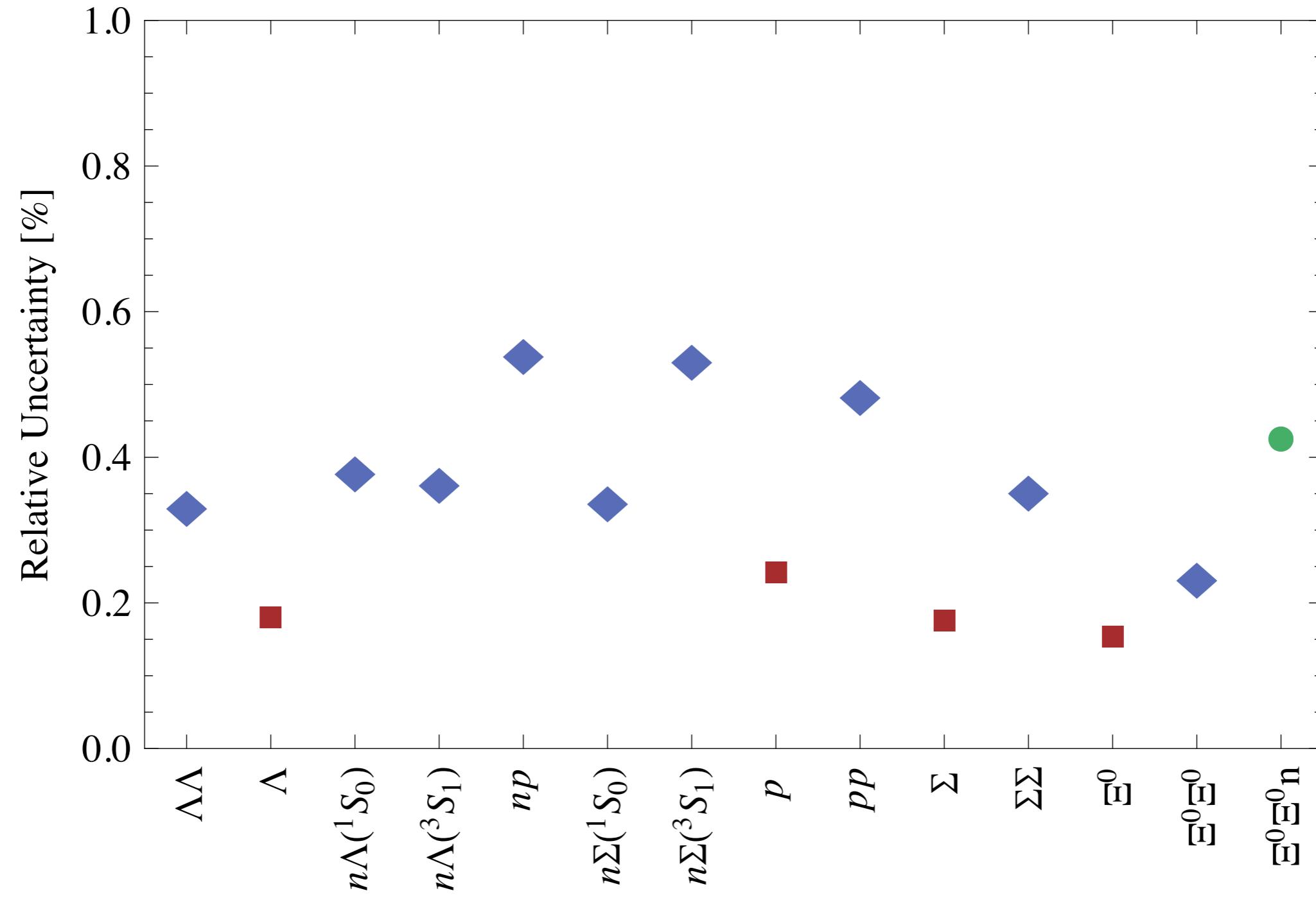


# Three Baryon Systems

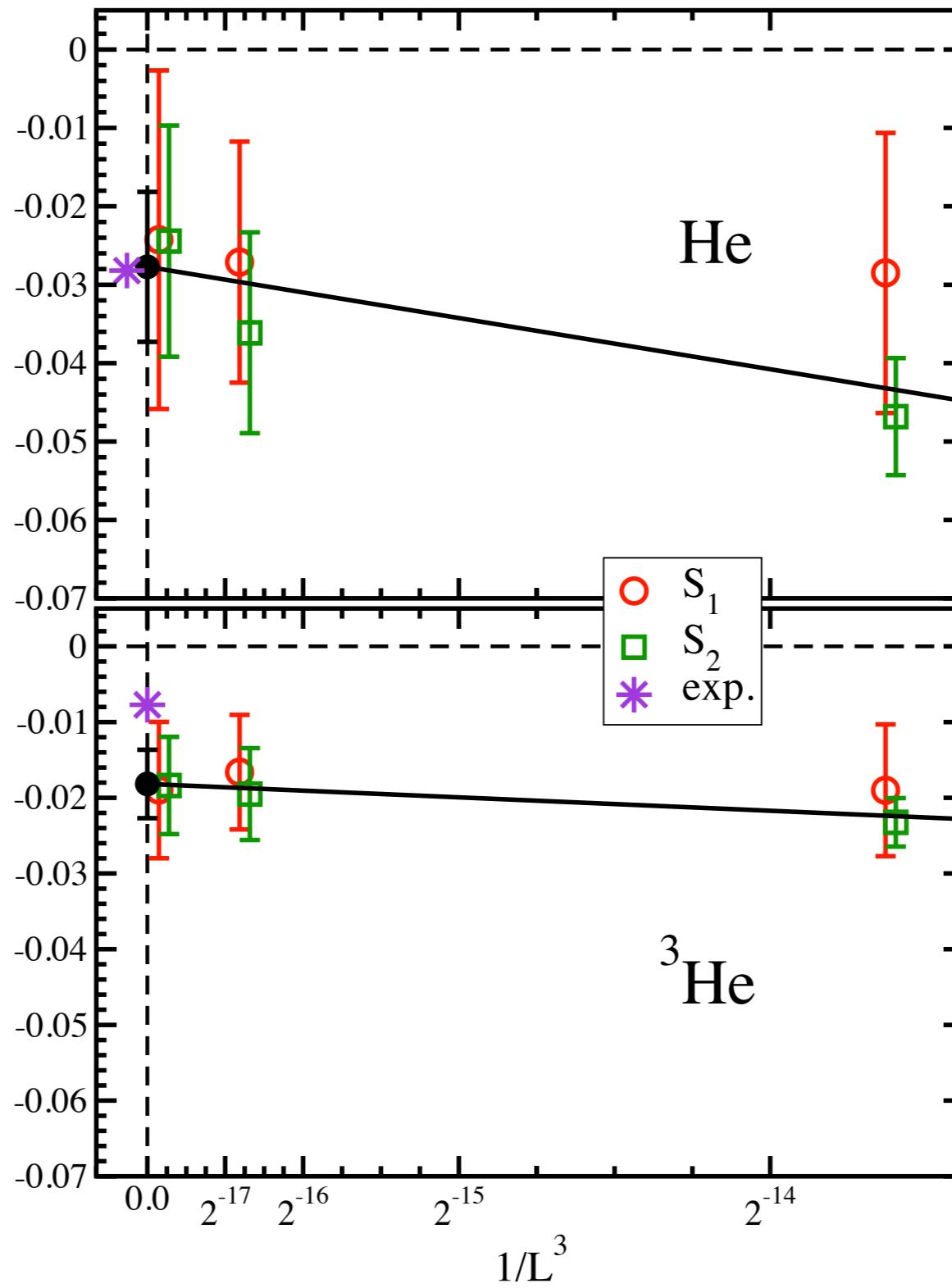




# Uncertainty per Baryon



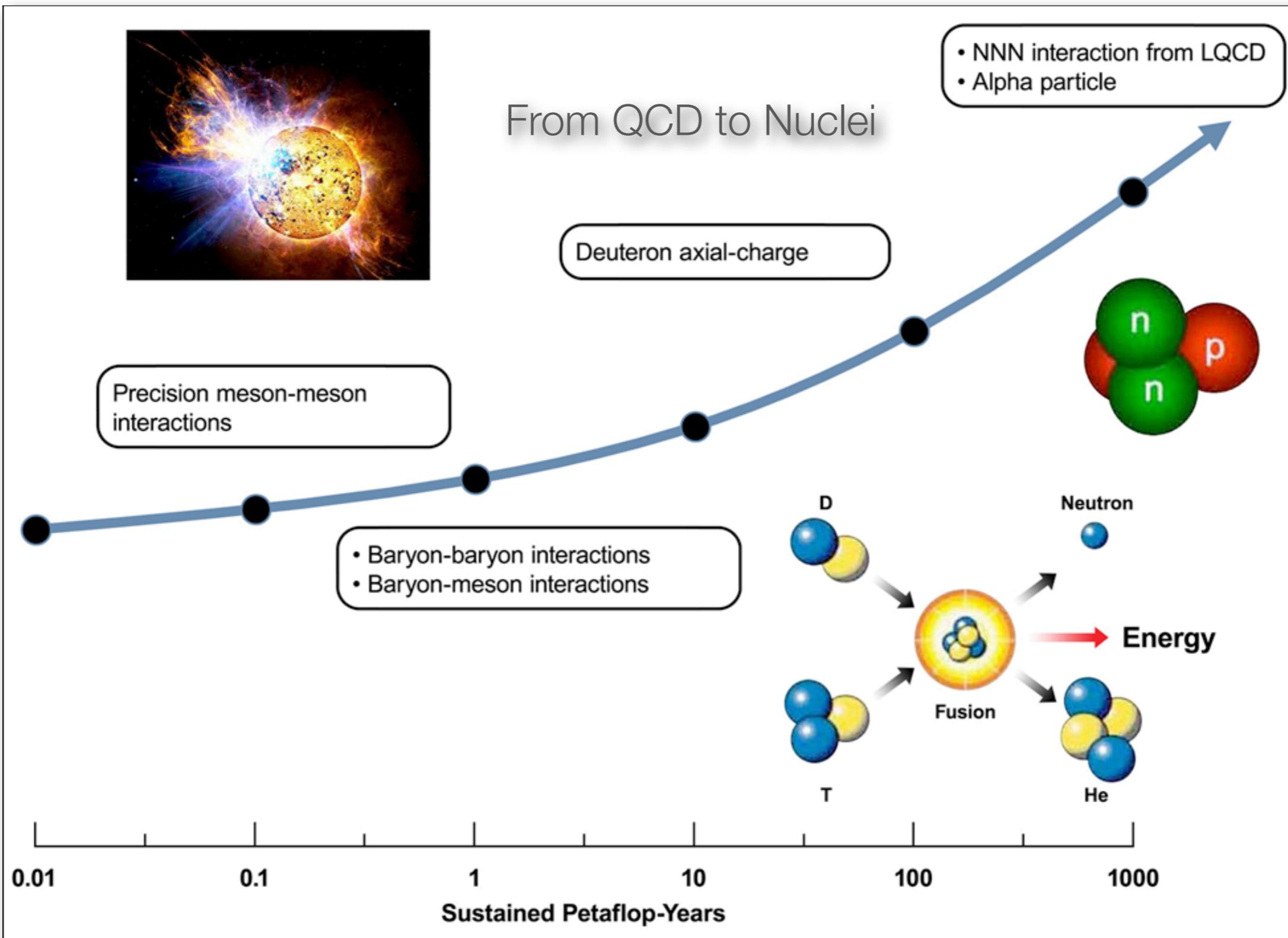
# Quenched A = 3 and 4 , $m_\pi \sim 800$ MeV



PACS-CS  
2009

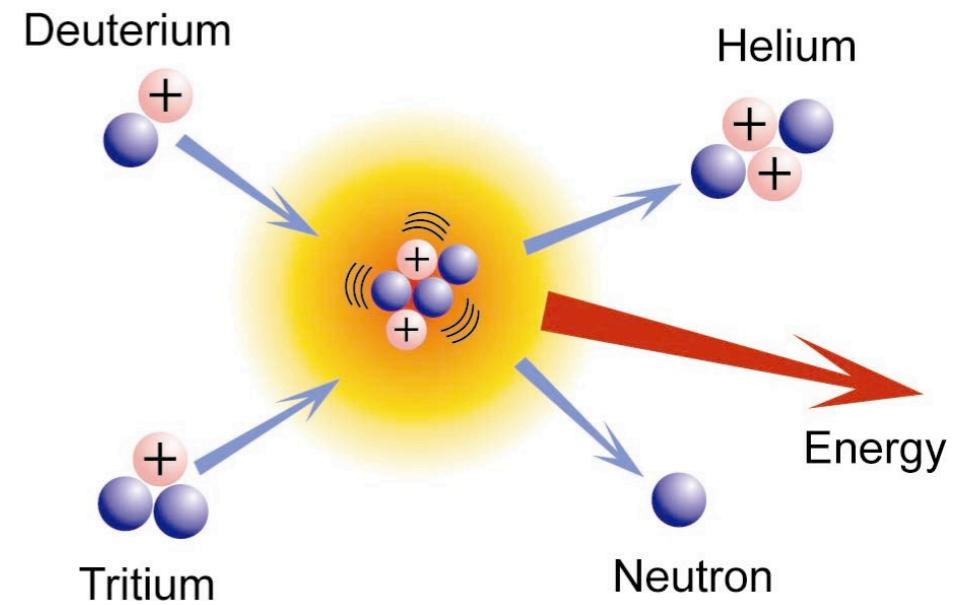


# Computational Requirements

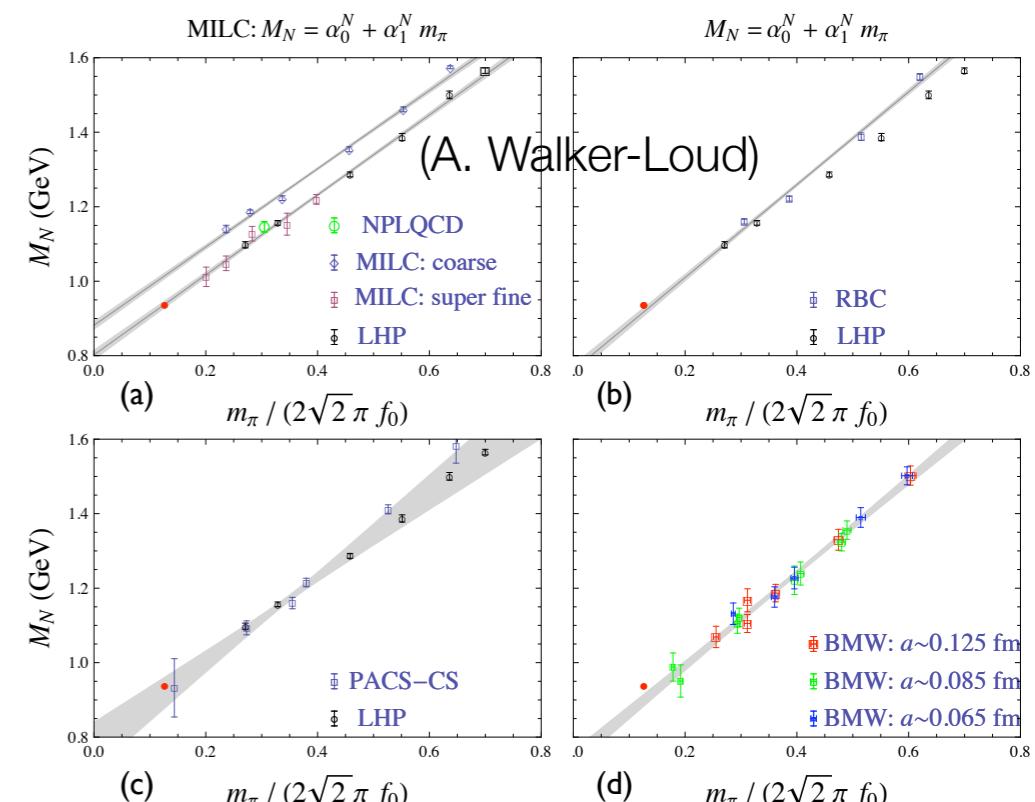


# Beyond Computational Requirements: Formal Issues , e.g.

What Lattice QCD calculations are required  
to predict multi-body nuclear reactions ?

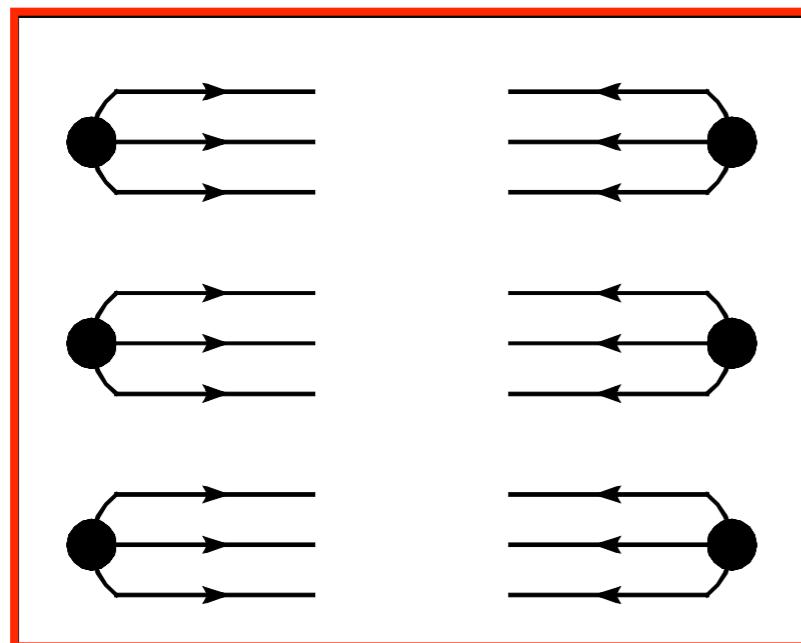


What length-scales determine the convergence  
of EFT expansions ?  
(to predict more complex systems than LQCD can access)  
(quark masses, number of flavors?)



# Many Nucleons (Baryons)

Large number of Wick contractions



$$\begin{aligned} \text{Proton : } N^{\text{cont}} &= 2 \\ ^{235}\text{U} : N^{\text{cont}} &= 10^{1494} \end{aligned}$$

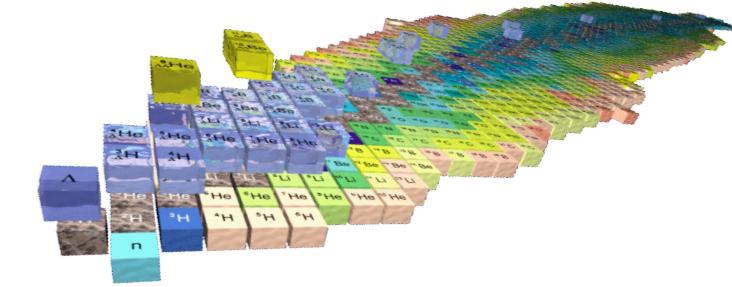
$$\begin{aligned} N_{\text{cont.}} &= u!d!s! \quad (\text{Naive}) \\ &= (A+Z)!(2A-Z)!s! \\ &\sim A^3 \quad (\text{Kaplan}) \end{aligned}$$

Symmetries provide significant reduction

$${}^3\text{He} : 2880 \rightarrow 93$$

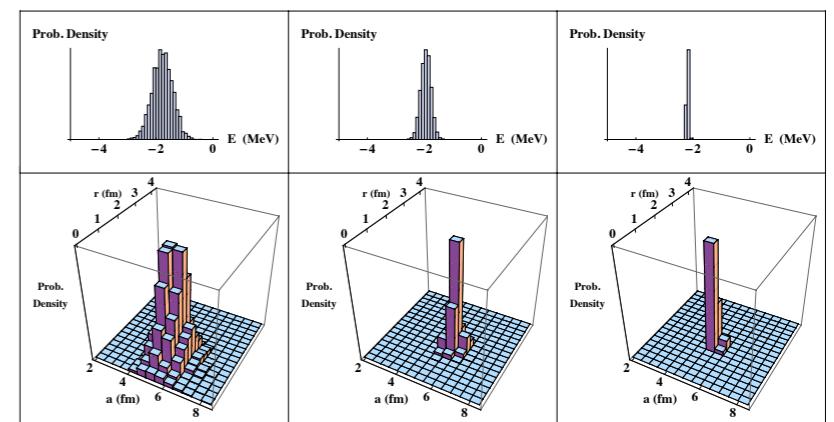
Recursion Relations

# Nuclear Theory Needs (physical $m_\pi$ )

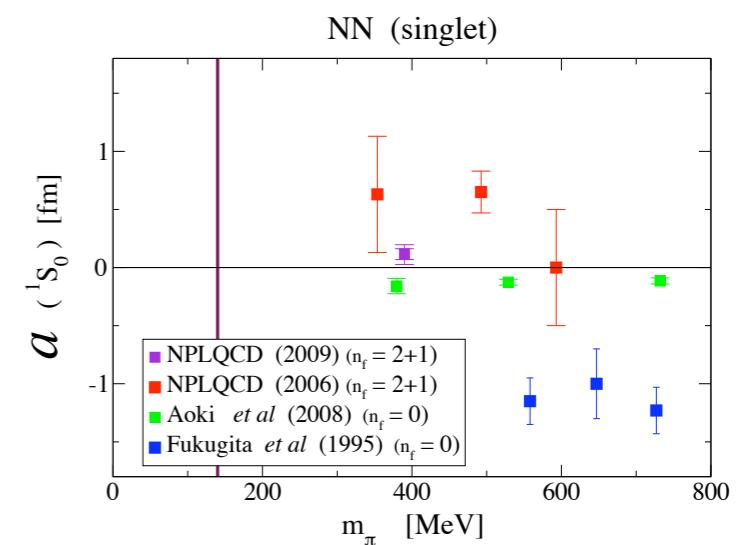


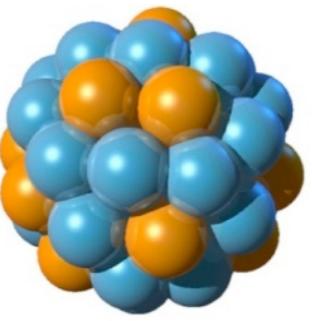
- What do we expect to observe in LQCD calculations in a finite volume
  - Need 3-body and 4-body spectrum (cubic volumes)
    - With and without background electroweak fields

- How to optimally invert lattice data
  - what volumes should we calculate with ?



- Quark-mass dependence
  - Higher orders in EFT for few-body systems





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# Lattice QCD is on the verge of making significant contributions to Low-Energy Nuclear Physics.

- The next several years will see remarkable things
- Lattice QCD calculations
  - at the physical pion mass,
  - in large volumes
  - at small lattice spacings

Nuclei and their interactions will be derived from QCD

END

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